Three Dimensional Geometry Class 12 Formula Sheet

CBSE Class 12 Maths · NCERT · for boards, NEET and JEE revision

ChapterNotes

1. Direction Cosines and Direction Ratios

If a line makes angles α, β, γ with the positive x, y, z axes respectively:
Direction Cosines: l = cos α, m = cos β, n = cos γ
Relation: $l^2 + m^2 + n^2 = 1$
Direction Ratios: a, b, c (proportional to l, m, n)
$l = a/\sqrt{(a^2+b^2+c^2)}$, $m = b/\sqrt{(a^2+b^2+c^2)}$, $n = c/\sqrt{(a^2+b^2+c^2)}$

2. Equation of a Line in 3D

Vector Form: Line passing through point A(a→) and parallel to vector b→:
$\mathbf{\vec{r} = \vec{a} + \lambda \vec{b}}$
Line through two points A(a→) and B(b→):
$\mathbf{\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})}$
Cartesian Form: Through point (x_1, y_1, z_1) with direction ratios a, b, c:
$\mathbf{(x - x_1)/a = (y - y_1)/b = (z - z_1)/c}$
Through two points (x_1, y_1, z_1) and (x_2, y_2, z_2):
$\mathbf{(x - x_1)/(x_2 - x_1) = (y - y_1)/(y_2 - y_1) = (z - z_1)/(z_2 - z_1)}$

3. Angle Between Two Lines

If lines have direction ratios (a_1, b_1, c_1) and (a_2, b_2, c_2):
$\mathbf{\cos\theta = |a_1a_2 + b_1b_2 + c_1c_2| / (\sqrt{(a_1^2+b_1^2+c_1^2)} \times \sqrt{(a_2^2+b_2^2+c_2^2)})}$
Parallel: $a_1/a_2 = b_1/b_2 = c_1/c_2$
Perpendicular: $a_1a_2 + b_1b_2 + c_1c_2 = 0$

4. Shortest Distance Between Two Lines

Skew Lines (non-parallel, non-intersecting): Lines: $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$
Shortest Distance $= \mathbf{|(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})| / |\vec{b_1} \times \vec{b_2}|}$
If shortest distance = 0, the lines **intersect**.
Parallel Lines: Lines: $\vec{r} = \vec{a_1} + \lambda\vec{b}$ and $\vec{r} = \vec{a_2} + \mu\vec{b}$
Distance $= \mathbf{|\vec{b} \times (\vec{a_2} - \vec{a_1})| / |\vec{b}|}$

5. Equation of a Plane

Form	Equation	When to Use
Normal form (vector)	$\vec{r} \cdot \hat{n} = d$	Given normal and distance from origin
General form	$ax + by + cz + d = 0$	Standard form; (a,b,c) is normal
Point-normal (vector)	$(\vec{r} - \vec{a}) \cdot \vec{n} = 0$	Given a point and normal
Point-normal (Cartesian)	$a(x-x_1) + b(y-y_1) + c(z-z_1) = 0$	Given point (x_1,y_1,z_1) and normal (a,b,c)
Intercept form	$x/a + y/b + z/c = 1$	Given x, y, z intercepts
Three-point form	Use determinant	Given 3 non-collinear points

6. Angle Between Two Planes

Planes: $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$
$\mathbf{\cos\theta = |a_1a_2 + b_1b_2 + c_1c_2| / (\sqrt{(a_1^2+b_1^2+c_1^2)} \times \sqrt{(a_2^2+b_2^2+c_2^2)})}$
Parallel: $a_1/a_2 = b_1/b_2 = c_1/c_2$
Perpendicular: $a_1a_2 + b_1b_2 + c_1c_2 = 0$

7. Distance of a Point from a Plane

Distance of point (x_1, y_1, z_1) from plane $ax + by + cz + d = 0$:
$\mathbf{D = |ax_1 + by_1 + cz_1 + d| / \sqrt{(a^2 + b^2 + c^2)}}$

8. Angle Between a Line and a Plane

Line: $\vec{r} = \vec{a} + \lambda\vec{b}$, Plane: $\vec{r} \cdot \vec{n} = d$
$\mathbf{\sin\varphi = |\vec{b} \cdot \vec{n}| / (|\vec{b}| \times |\vec{n}|)}$

Quick revision

- Direction cosines: $l^2 + m^2 + n^2 = 1$ (always!)
- Line equation: $\vec{r} = \vec{a} + \lambda\vec{b}$ or $(x-x_1)/a = (y-y_1)/b = (z-z_1)/c$
- Plane equation: $\vec{r} \cdot \vec{n} = d$ or $ax + by + cz + d = 0$
- Parallel lines ⟹ proportional direction ratios
- Perpendicular lines ⟹ $a_1a_2 + b_1b_2 + c_1c_2 = 0$
- Distance from plane: $|ax_1+by_1+cz_1+d| / \sqrt{(a^2+b^2+c^2)}$
- Skew lines: use cross product formula for shortest distance
- Line-plane angle uses sin, plane-plane angle uses cos

Full chapter notes with solved examples and practice questions: https://chapternotes.in/cbse/class-12-maths/three-dimensional-geometry-class-12-notes/