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CHAPTER NOTES

Real Numbers Class 10 Notes | CBSE Maths Chapter 1

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Ask me anything from this chapter. A concept, a numerical, or that one doubt you did not want to raise in class. Pick one to start:

Explain Real Numbers simply

What can the board exam ask?

Most important formulas

Quiz me on this chapter

Ask Ira anything...



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Real Numbers Class 10 Notes | CBSE Maths Chapter 1

Real Numbers is Chapter 1 of CBSE Class 10 Maths. It picks up the real numbers you met in Class IX and looks at them more closely. The chapter covers the Fundamental Theorem of Arithmetic, finding the HCF and LCM of numbers by prime factorisation, the relation $\text{HCF} \times \text{LCM} = \text{product of two numbers}$, and proofs that numbers such as $\sqrt{2}$, $\sqrt{3}$ and $\sqrt{5}$ are irrational.

For board exams the chapter is short and very predictable. Questions ask you to factorise a number into primes, find an HCF or LCM, use the HCF and LCM relation, decide whether a number such as 6^n can end in zero, solve a word problem on bells or runners meeting again, and write a full proof by contradiction that a number is irrational. Every one of these rests on a single idea: each composite number has exactly one prime factorisation.

Quick Answer - Real Numbers in 3

The **Fundamental Theorem of Arithmetic** states that every integer greater than 1 is either a prime or a product of primes in exactly one way, up to the order of the factors. For two numbers a and b , $\text{HCF} \times \text{LCM} = a \times b$. For three numbers a , b , and c , $\text{HCF} \times \text{LCM}(a, b) = a \times b$; for three numbers a , b , and c , then p divides a . Using this, a proof by contradiction that every prime p are irrational.



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All Key Formulas at a Glance

Idea	Formula or rule	Use it when
Fundamental Theorem of Arithmetic	Composite $x = p_1 p_2 \dots p_n$, unique apart from order	Factorising a number, or arguing which primes can appear
HCF by prime factorisation	Product of the smallest power of each common prime factor	Finding the HCF of two or more numbers
LCM by prime factorisation	Product of the greatest power of each prime factor involved	Finding the LCM of two or more numbers
HCF and LCM relation	$HCF(a, b) \times LCM(a, b) = a \times b$	Two numbers only: finding LCM from HCF, or a missing
LCM of three numbers	$LCM(p, q, r) = \frac{p \times q \times r}{HCF(p, q) \times HCF(p, r)}$	
HCF of three numbers	$HCF(p, q, r) = \frac{p \times q \times r}{LCM(p, q) \times LCM(p, r)}$	
Theorem 1.2	p prime and p divides a	
Irrational number	Cannot be written as $\frac{p}{q}$ where p, q are integers and $q \neq 0$	
Rational and irrational combined	Rational $\pm$ irrational = irrational Non-zero rational $\times$ irrational = irrational	



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Key Concepts

1. Primes, Composites and the Fundamental Theorem of Arithmetic

A **prime number** is a natural number greater than 1 whose only factors are 1 and itself: 2, 3, 5, 7, 11, 13, 17, 19, 23 and so on. A **composite number** is a natural number greater than 1 that has at least one more factor: 4, 6, 8, 9, 10, 12 and so on. The number 1 is neither prime nor composite. The number 2 is the only even prime.

The NCERT chapter first works in the other direction. Multiply some primes, say 2, 3, 7, 11 and 23, repeating them as often as you like, and you get numbers such as $7 \times 11 \times 23 = 1771$ and $2^3 \times 3 \times 7^3 = 8232$.

There are infinitely many primes, so combining all of them in all possible ways gives an infinite collection of numbers. The question the chapter asks is whether every composite number can be produced this way. The answer is yes, and it comes with a second promise: each composite number is produced in only one way.

Breaking a number down with a factor tree

To factorise 32760, keep splitting off the smallest prime factor.

Step	Divide by
1	2
2	2
3	2
4	3
5	3
6	5
7	7

So $32760 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7 \times 13$
bigger case: $123456789 = 3^2 \times 3803 \times 36$



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The theorem

Theorem 1.1 (Fundamental Theorem of Arithmetic): Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.

The theorem makes two separate claims:

1. **Existence:** every composite number can be written as a product of primes.
2. **Uniqueness:** there is only one such product. $2 \times 3 \times 5 \times 7$ and $3 \times 5 \times 7 \times 2$ count as the same factorisation, because only the order differs.

The same fact is often stated as: **the prime factorisation of a natural number is unique, except for the order of its factors.** If we agree to write the primes in ascending order, $p_1 \leq p_2 \leq \dots \leq p_n$, the factorisation becomes completely fixed. Grouping equal primes gives powers of primes, as in $2^3 \times 3^2 \times 5 \times 7 \times 13$.

History: an equivalent version was probably known to the ancient Greeks in Euclid's Elements; the first correct proof was given by Eratosthenes.

2. HCF and LCM by Prime Factorisation

The **HCF** (highest common factor) of two numbers is the largest number that divides each of them. The **LCM** (lowest common multiple) is the smallest number that each of them divides. Once the numbers are written in their prime factorised form, we can find HCF and LCM directly. This is called the **prime factorisation method**.

- **HCF** = product of the **smallest power** of common prime factors
- **LCM** = product of the **greatest power** of common prime factors

NCERT Example 2 (6 and 20): $6 = 2^1 \times 3^1$ and $20 = 2^2 \times 5^1$. The smaller power is 2^1 , so $\text{HCF}(6, 20) = 2$. The greater power is 2^2 , 3^1 , 5^1 , so $\text{LCM}(6, 20) = 2^2 \times 3 \times 5 = 60$.

Laying out the work in a table keeps you organised. For example, for 7 and 54: $7 = 7^1$ and $54 = 2 \times 3^3$.



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Prime	Power in 336	Power in 54	For HCF (smaller)	For LCM (greater)
2	4	1	2^1	2^4
3	1	3	3^1	3^3
7	1	0	not common, left out	7^1

HCF = $2 \times 3 = 6$ and LCM = $2^4 \times 3^3 \times 7 = 16 \times 27 \times 7 = 3024$.

Three or more numbers work the same way; see Example 6 for 6, 72 and 120.

Two quick checks catch most errors: the HCF must divide every number, and every number must divide the LCM. Also, $\text{HCF} \leq \text{smallest number}$ and $\text{LCM} \geq \text{largest number}$.

Coprime numbers. Two numbers are coprime if their HCF is 1. For 17, 23 and 29, all primes, the HCF is 1 and LCM = $17 \times 23 \times 29 = 11339$.

3. The Relation $\text{HCF} \times \text{LCM} = \text{Product}$

For 6 and 20: $\text{HCF} \times \text{LCM} = 2 \times 60 = 120$, which is the product of the two positive integers a and b,

$$\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$$

Why it works. For each prime, the HCF takes the smaller power and the LCM takes the greater. Together they use both powers.

Using the relation. Once the HCF is known, the LCM can be found by

$$\text{LCM}(a, b) = (a \times b) / \text{HCF}(a, b)$$

NCERT Example 3: $96 = 2^5 \times 3$ and $404 = 2^2 \times 101$.
 $\text{HCF} = 2^2 = 4$, $\text{LCM} = (96 \times 404) / 4 = 96 \times 101 = 9696$.

It does not hold in general for three numbers. For 6, 15 and 20, $\text{HCF} \times \text{LCM} = 3 \times 60 = 180$, while $\text{HCF} \times \text{LCM} = 6 \times 360 = 2160$.



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triples (for 8, 9 and 25, $\text{HCF} = 1$ and $\text{LCM} = 1800 = 8 \times 9 \times 25$), so it is never a rule you can use for three numbers.

What does hold for three numbers. NCERT's note to the reader gives these results for positive integers p, q and r :

- $\text{LCM}(p, q, r) = p \cdot q \cdot r \cdot \text{HCF}(p, q, r) / [\text{HCF}(p, q) \cdot \text{HCF}(q, r) \cdot \text{HCF}(p, r)]$
- $\text{HCF}(p, q, r) = p \cdot q \cdot r \cdot \text{LCM}(p, q, r) / [\text{LCM}(p, q) \cdot \text{LCM}(q, r) \cdot \text{LCM}(p, r)]$

Question 15 in the board questions below checks the LCM formula on 144, 180 and 192.

A consequence worth remembering: the HCF of two numbers always divides their LCM. So a pair with HCF 18 and LCM 380 cannot exist, because $380 \div 18$ is not a whole number.

4. Applications of Unique Factorisation

The uniqueness part of the theorem turns which primes are present.

(a) Can a power end in the digit 0?

A number ends in 0 exactly when it is divisible by 10. This means it must contain **both 2 and 5**.

Number	Prime factorisation
4^n	2^{2n}
6^n	$2^n \times 3^n$
12^n	$2^{2n} \times 3^n$
15^n	$3^n \times 5^n$
20^n	$2^{2n} \times 5^n$

The written argument must mention unique factorisation. By the Fundamental Theorem of Arithmetic, there are no other

(b) Showing a number is composite



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Take out a common factor: $7 \times 11 \times 13 + 13 = 13 \times 78$, so it has a factor other than 1 and itself (see Example 8).

(c) Meeting again: an LCM word problem

When events repeat at fixed intervals and you want the first time they happen together again, find the LCM of the intervals. Bells, traffic lights, runners on a track and buses leaving a depot all fit this pattern.

(d) Largest equal groups: an HCF word problem

When you split several quantities into the largest possible equal groups, with nothing left over, find the HCF. Rows of students, stacks of sweets, lengths of rope cut into equal pieces and square tiles covering a floor fit this pattern.

Key words in the question	Use
"meet again", "ring together", "least time", "divisible by"	
"maximum number of rows", "largest size", "each"	



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5. Theorem 1.2: If a Prime Divides a

Theorem 1.2: Let p be a prime number. If p divides a^2 , then p divides a .

Proof idea (NCERT marks this proof as not for understanding it makes the irrationality proof not necessarily distinct. Then $a^2 = p_1^2 p_2^2 \dots p_k^2$ of a^2 . By uniqueness, the only prime factor of a^2 is p , therefore p divides a .

Illustration. $225 = 15^2 = 3^2 \times 5^2$. Both 3 and 5 are prime, so 3 and 5 divide 15. Bringing in no new one.

The theorem needs p to be prime. 4 divides 16, but 4 is not prime. If 4 divides a composite, the theorem says nothing about it. Only when you use Theorem 1.2.

6. Proving that $\sqrt{2}$, $\sqrt{3}$ and $\sqrt{5}$ Are Irrational

A number s is called **irrational** if it cannot be written in the form p/q , where p and q are integers and $q \neq 0$. Familiar examples are $\sqrt{2}$, $\sqrt{3}$, $\sqrt{15}$, π and $0.1011011101110\dots$

In Class IX you used these numbers without a proof that they are irrational. This chapter proves it using **proof by contradiction**: assume the opposite of what you want, reason correctly, reach something impossible, and conclude that the assumption was wrong.

The proof that $\sqrt{2}$ is irrational (Theorem 1.3), step by step

1. **Assume the opposite.** Suppose $\sqrt{2}$ is rational. Then $\sqrt{2} = r/s$ for integers r and s , $s \neq 0$.
2. **Reduce to lowest terms.** If r and s have a common factor other than 1, divide it out to get $\sqrt{2} = a/b$, where a and b are **coprime**.
3. **Clear the root.** $b\sqrt{2} = a$. Squaring, $2b^2 = a^2$. So 2 divides a^2 .
4. **Use Theorem 1.2.** Since 2 is prime and divides a^2 , it divides a . So $a = 2c$ for some integer c .
5. **Substitute back.** $2b^2 = 4c^2$, so $b^2 = 2c^2$. So 2 divides b^2 .
6. **Contradiction.** Now 2 divides both a and b , which contradicts the fact that a and b are coprime.
7. **Conclude.** The contradiction arose from assuming $\sqrt{2}$ is rational. Therefore, $\sqrt{2}$ is irrational.

The same template works for any prime p (to prove $\sqrt{p}$ is irrational): $3b^2 = a^2$, so 3 divides a , $a = 3c$, and you get Exercise 1.2 Question 1. The conclusion is that $\sqrt{p}$ is irrational whenever p is a prime.



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Step	For $\sqrt{p}$ (p prime)
Assume	$\sqrt{p} = a/b$, a and b coprime, $b \neq 0$
Square	$p b^2 = a^2$
Theorem 1.2	p divides a, so $a = pc$
Substitute	$p b^2 = p^2 c^2$, so $b^2 = p c^2$
Theorem 1.2 again	p divides b
Contradiction	p is a common factor of a and b

7. Sums, Differences, Products and

The chapter uses these facts from Class 10

- The sum or difference of a rational number and an irrational number is irrational.
- The product and quotient of a **non-zero** rational number and an irrational number is irrational. ($0 \times \sqrt{2} = 0$ is rational).

To prove a particular case such as $5 - \sqrt{3}$ is irrational, this time the contradiction is with a number being rational.

1. Assume the number is rational, equal to a/b .
2. Rearrange to get the known irrational alone on one side.
3. The other side is built from integers only.
4. So the known irrational would be rational.

NCERT Example 6: if $5 - \sqrt{3} = a/b$, then $\sqrt{3} = (5b - a)/b$. Here a and b are integers. This contradicts the fact that $\sqrt{3}$ is irrational.

NCERT Example 7: if $3\sqrt{2} = a/b$, then $\sqrt{2} = a/(3b)$. Here a and b are integers. This contradicts the fact that $\sqrt{2}$ is irrational.

A caution. Two irrationals can combine to give a rational. The rules above cover a rational mixed with an irrational.



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Formula and Theorem Sheet

Result	Statement	Remember
Theorem 1.1 (FTA)	Every composite number can be expressed as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur	Two parts: existence and uniqueness
HCF	Product of the smallest power of each common prime factor	Only primes common to all the numbers
LCM	Product of the greatest power of each prime factor involved	Every prime that appears anywhere
Two-number relation	$HCF(a, b) \times LCM(a, b)$	
Three numbers	$HCF(p, q, r) \times LCM(p, q, r)$	
LCM of three numbers	$p \cdot q \cdot r \cdot HCF(p, q, r) / HCF(p, r)$	
HCF of three numbers	$p \cdot q \cdot r \cdot LCM(p, q, r) / LCM(p, r)$	
Theorem 1.2	p prime, p divides a^2 $\Rightarrow$ p divides a (where a is an integer)	
General result	$\sqrt{p}$ is irrational for every prime p	
Ends in 0	Number divisible by 10 is divisible by both 2 and 5	



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Important Definitions

- **Prime number:** a natural number greater than 1 whose only factors are 1 and the number itself.
- **Composite number:** a natural number greater than 1 that has at least one factor other than 1 and itself.
- **Fundamental Theorem of Arithmetic:** every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.
- **HCF (Highest Common Factor):** the greatest positive integer that divides each of the given numbers.
- **LCM (Lowest Common Multiple):** the smallest positive integer that is divisible by each of the given numbers.
- **Coprime numbers:** two integers whose only common factor is 1, that is, their HCF is 1.
- **Rational number:** a number that can be written as $\frac{p}{q}$, where p and q are integers and $q \neq 0$.
- **Irrational number:** a number that cannot be written as $\frac{p}{q}$, where p and q are integers and $q \neq 0$.
- **Proof by contradiction:** a method of proving a statement is true by assuming it is false and showing it leads to a contradiction, or a statement that is both true and false.



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Solved Examples (NCERT)

Example 1: Express each number as a product of prime factors. (i) 140 (ii) 156 (iii) 3825 (iv) 5005 (v) 7429 (E)

(i) $140 = 2 \times 70 = 2 \times 2 \times 35 = 2 \times 2 \times 5 \times 7$

(ii) $156 = 2 \times 78 = 2 \times 2 \times 39 = 2 \times 2 \times 3 \times 13$

(iii) 3825 ends in 5: $3825 = 5 \times 765 = 5 \times 5 \times 153 = 5 \times 5 \times 3 \times 17$. So $3825 = 3^2 \times 5^2 \times 17$.

(iv) $5005 = 5 \times 1001 = 5 \times 7 \times 143 = 5 \times 7 \times 11 \times 13$

(v) 7429 is not divisible by 2, 3, 5, 7, 11 or 13. $7429 = 17 \times 19 \times 23$.

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Example 2: Consider the numbers 4^n , where n is a natural number. Check whether there is any value of n for which 4^n ends with the digit zero. (NCERT Example 1)

If 4^n ended with the digit zero for some n , it would be divisible by 5. Then the prime factorisation of 4^n would contain the prime 5.

But $4^n = (2^2)^n = 2^{2n}$, so the only prime in its factorisation is 2.

By the uniqueness of the Fundamental Theorem of Arithmetic, there are no other primes in the factorisation of 4^n . So 5 does not divide 4^n .

$\therefore$ There is no natural number n for which 4^n ends with the digit zero.

Example 3: Check whether 6^n can end with the digit 0 for any natural number n . (Exercise 1.1, Q5)

For 6^n to end with 0, it must be divisible by 5.

$6^n = (2 \times 3)^n = 2^n \times 3^n$. The only primes are 2 and 3.

By the uniqueness of the Fundamental Theorem of Arithmetic, 5 does not divide 6^n .

$\therefore 6^n$ cannot end with the digit 0 for any natural number n .

Example 4: Find the LCM and HCF product of the two numbers. (Exercise 1.4, Q1)

$$510 = 2 \times 3 \times 5 \times 17$$

$$92 = 2^2 \times 23$$

The only common prime is 2; its smaller power is 2^1 .

Greatest powers of all primes involved: $2^2, 3, 5, 17, 23$.

Verification: $\text{LCM} \times \text{HCF} = 23460 \times 2 = 46920$, which is equal to the product of the two numbers, as required.



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Example 5: Find the LCM and HCF of 12, 15 and 21 by applying the prime factorisation method. (Exercise 1.1, Q3(i))

$$12 = 2^2 \times 3, 15 = 3 \times 5, 21 = 3 \times 7$$

The only prime common to all three is 3, with smallest power 3^1 . **HCF = 3.**

Greatest powers of all primes involved: $2^2, 3^1, 5^1, 7^1$. **LCM = $4 \times 3 \times 5 \times 7 = 420$.**

Check: $420 \div 12 = 35$, $420 \div 15 = 28$, $420 \div 21 = 20$, all whole numbers.

Example 6: Find the HCF and LCM of 6, 72 and 120, using the prime factorisation method. (NCERT Example 4)

$$6 = 2 \times 3, 72 = 2^3 \times 3^2, 120 = 2^3 \times 3 \times 5$$

2^1 and 3^1 are the smallest powers of the common factors 2 and 3. So **HCF(6, 72, 120) = $2 \times 3 = 6$.**

$2^3, 3^2$ and 5^1 are the greatest powers of the prime factors. So **LCM = $2^3 \times 3^2 \times 5 = 360$.**

Remark: $6 \times 72 \times 120 = 51840$, while HCF of the numbers is not equal to the product of the numbers.

Example 7: Given that HCF (306, 657) = 9, find their LCM. (Exercise 1.4, Q4)

For two positive integers, $\text{HCF} \times \text{LCM} = \text{product of the numbers}$

$$\text{LCM}(306, 657) = (306 \times 657) / 9$$

$$306 \div 9 = 34, \text{ so } \text{LCM} = 34 \times 657 = \mathbf{22338}$$

Check with factorisations: $306 = 2 \times 3^2 \times 17$, $657 = 3^2 \times 73$
 $= 2 \times 3^2 \times 17 \times 73 = 18 \times 1241 = 22338$.

Example 8: Explain why $7 \times 11 \times 13$ is not a composite number. (Exercise 1.1, Q5)

First number: $7 \times 11 \times 13 + 13 = 13 \times (7 \times 11 + 1)$

1014 has the factor 13 besides 1 and itself. So it is composite. In fact $1014 = 2 \times 3 \times 13^2$.



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Second number: $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5 = 5 \times (7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1) = 5 \times (1008 + 1) = 5 \times 1009 = 5045$.

5045 has the factor 5 besides 1 and itself. So it is composite.

Example 9: There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point? (Exercise 1.1, Q7)

Sonia is at the starting point after 18, 36, 54, ... minutes (multiples of 18). Ravi is there after 12, 24, 36, ... minutes (multiples of 12). They meet at the starting point at the first common multiple, which is the LCM.

$$18 = 2 \times 3^2 \text{ and } 12 = 2^2 \times 3$$

$$\text{LCM} = 2^2 \times 3^2 = 36$$

∴ They meet again at the starting point after 36 minutes, which is after Ravi 3 rounds.

Example 10: Prove that $\sqrt{5}$ is irrational.

Let us assume, to the contrary, that $\sqrt{5}$ is rational.

Then we can find integers a and b ($b \neq 0$) such that $\sqrt{5} = \frac{a}{b}$. If b is other than 1, we divide by it, so we may assume that b = 1.

So $b\sqrt{5} = a$. Squaring both sides, $5b^2 = a^2$.

Since 5 is prime, by Theorem 1.2, 5 divides a^2 .

Substituting, $5b^2 = 25c^2$, that is, $b^2 = 5c^2$.

So a and b have at least 5 as a common factor. This contradicts the fact that a and b are coprime.

This contradiction arose from our incorrect assumption that $\sqrt{5}$ is rational.



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Example 11: Prove that $3 + 2\sqrt{5}$ is irrational. (Exercise 1.2, Q2)

Let us assume, to the contrary, that $3 + 2\sqrt{5}$ is rational.

Then we can find coprime integers a and b ($b \neq 0$) such that $3 + 2\sqrt{5} = a/b$.

Rearranging: $2\sqrt{5} = a/b - 3 = (a - 3b)/b$, so $\sqrt{5} = (a - 3b)/(2b)$.

Since a and b are integers, $a - 3b$ and $2b$ are integers and $2b \neq 0$. So $(a - 3b)/(2b)$ is rational, and hence $\sqrt{5}$ is rational.

This contradicts the fact that $\sqrt{5}$ is irrational (Example 10).

$\therefore 3 + 2\sqrt{5}$ is irrational.

Example 12: Prove that the following are irrationals: (i) $1/\sqrt{2}$ (ii) $7\sqrt{5}$ (iii) $6 + \sqrt{2}$ (Exercise 1.2, Q3)

(i) Assume, to the contrary, that $1/\sqrt{2}$ is rational. Since $1/\sqrt{2} \neq 0$, $a \neq 0$ as well. Taking reciprocal, b/a is rational, so $\sqrt{2}$ is rational. This contradicts the fact that $\sqrt{2}$ is irrational.

(ii) Assume, to the contrary, that $7\sqrt{5} = a/b$. Since 7 , a and b are integers and $7b \neq 0$, $a/(7b)$ is rational, so $\sqrt{5}$ is rational. This contradicts the fact that $\sqrt{5}$ is irrational. So $7\sqrt{5}$ is irrational.

(iii) Assume, to the contrary, that $6 + \sqrt{2} = a/b$. Since $a/b - 6 = (a - 6b)/b$. Since a and b are integers and $b \neq 0$, $(a - 6b)/b$ is rational, so $\sqrt{2}$ is rational. This contradicts the fact that $\sqrt{2}$ is irrational. So $6 + \sqrt{2}$ is irrational.



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Competency-Based Questions

For every assertion-reason item below, choose the correct option.

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Assertion (A) is false, but Reason (R) is true.

Free questions, no login needed

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1. Case-based: The school bells

A school has three electronic bells. The first rings every 6 minutes, the second every 8 minutes and the third every 12 minutes. All three ring together at 8:00 a.m.

(a) Write the prime factorisations of 6, 8 and 12. $6 = 2 \times 3$, $8 = 2^3$, $12 = 2^2 \times 3$.

(b) When will they next ring together? $\text{LCM} = 2^3 \times 3 = 24$ minutes, so at 8:24 a.m.

(c) How many times will they ring together after 8:00 a.m., up to and including 10:00 a.m.? $120 \text{ minutes} \div 24 = 5$. They ring together at 8:24, 8:48, 9:12, 9:36 and 10:00, that is, **5 times**.

2. Case-based: The marching contingent

An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns.

(a) Which idea gives the maximum number of columns? The number of columns must divide both 616 and 32, and we want the maximum number.

(b) Find it. $616 = 2^3 \times 7 \times 11$ and $32 = 2^5$. The common factor is $2^3 = 8$.

(c) How many rows will each group then have? The number of rows will be the same as the number of columns, which is 8.

3. Case-based: The sweet shop

A sweet seller has 420 kaju barfis and 130 laddoos. She wants to put them in separate stacks so that each stack has the same number of sweets and the number of stacks is as large as possible. How many stacks will she have?

(a) How many barfis should go in each stack? The number of barfis in each stack should be as large as possible. $420 = 2^2 \times 3 \times 5 \times 7$ and $130 = 2 \times 5 \times 13$. The common factor is $2 \times 5 = 10$.

(b) How many stacks are there in all? $420 \div 10 = 42$ and $130 \div 10 = 13$. So there are $42 + 13 = 55$ stacks in all.

4. Source-based: Reading a proof

Read this student's proof: "Suppose $\sqrt{3} = \frac{a}{b}$, where a and b are coprime. Then $3b^2 = a^2$. So 3 divides a^2 , and hence 3 divides a . Let $a = 3c$. Then $3b^2 = (3c)^2 = 9c^2$. So $b^2 = 3c^2$. So 3 divides b^2 , and hence 3 divides b . This contradicts the fact that a and b are coprime. Hence $\sqrt{3}$ is irrational."



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(a) Which theorem justifies “3 divides a^2 , and hence 3 divides a ”? Theorem 1.2: if a prime p divides a^2 , then p divides a . It applies because 3 is prime.

(b) Fill in the missing working between “Let $a = 3c$ ” and “ $b^2 = 3c^2$ ”. Substituting $a = 3c$ in $3b^2 = a^2$ gives $3b^2 = 9c^2$, and dividing by 3 gives $b^2 = 3c^2$.

(c) What exactly does the contradiction contradict? The starting assumption that a and b are coprime, that is, have no common factor other than 1.

5. Assertion-Reason

Assertion (A): $\text{HCF}(6, 20) \times \text{LCM}(6, 20) = 120$.

Reason (R): For any two positive integers a and b , $\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$.

Answer: (A). $\text{HCF} = 2$, $\text{LCM} = 60$ and $2 \times 60 = 120 = 6 \times 20$, so A is true. R is the general result that gives A directly.

6. Assertion-Reason

Assertion (A): 6^n never ends with the digit 5.

Reason (R): The prime factorisation of 6^n is $2^n \times 3^n$.

Answer: (A). Ending in 0 needs the prime factors 2 and 5, which rules out a 5, which is exactly why A holds.

7. Assertion-Reason

Assertion (A): For the numbers 6, 72 and 108, $\text{HCF} \times \text{LCM} = \text{product of the numbers}$.

Reason (R): For any two positive integers a and b , $\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$.

Answer: (D). $\text{HCF} \times \text{LCM} = 6 \times 360 = 2160$, which is not equal to $6 \times 72 \times 108$, but it is guaranteed only for two numbers.

8. Assertion-Reason

Assertion (A): $7 \times 11 \times 13 + 13$ is a composite number.

Reason (R): The prime factorisation of a number is unique.



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Answer: (B). A is true because the number equals 13×78 . R is true, but A follows from taking out the common factor 13; uniqueness of factorisation is not the reason it is composite.

9. Assertion-Reason

Assertion (A): $2 + \sqrt{3}$ is an irrational number.

Reason (R): The product of any two irrational numbers is irrational.

Answer: (C). A is true: the sum of the rational 2 and the irrational $\sqrt{3}$ is irrational. R is false: $\sqrt{3} \times \sqrt{3} = 3$, which is rational.

Important Questions for Board Exams

1-Mark Questions

Q1. If $a = x^3y^2$ and $b = xy^3$, where x and y

Answer: HCF takes the smaller powers: x^1y^2

Q2. The HCF of two numbers is 13 and the

Answer: Other number = $(13 \times 182)/26 = 91$

Q3. Find the HCF of the smallest prime nu

Answer: Smallest prime = 2, smallest com

Q4. The product of a non-zero rational nu
always irrational (c) rational or irrational (d)

Answer: (b) always irrational.

Q5. Is $3 \times 5 \times 7 + 7$ prime or composite? C

Answer: $3 \times 5 \times 7 + 7 = 7 \times (15 + 1) = 7 \times 16$
composite.



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2-Mark Questions

Q6. Find the LCM and HCF of 26 and 91 and verify that $\text{LCM} \times \text{HCF} = \text{product of the two numbers}$.

Answer: $26 = 2 \times 13$ and $91 = 7 \times 13$. $\text{HCF} = 13$. $\text{LCM} = 2 \times 7 \times 13 = 182$. $\text{LCM} \times \text{HCF} = 182 \times 13 = 2366$, and $26 \times 91 = 2366$. Verified.

Q7. Check whether 12^n can end with the digit 0 for any natural number n .

Answer: $12^n = (2^2 \times 3)^n = 2^{2n} \times 3^n$. To end in 0 it must have 5 as a prime factor. By the uniqueness of the Fundamental Theorem of Arithmetic, its only primes are 2 and 3. So 12^n **cannot** end with 0.

Q8. Find the LCM and HCF of 8, 9 and 25 by applying the prime factorisation method.

Answer: $8 = 2^3$, $9 = 3^2$, $25 = 5^2$. No prime is common to all three. $\text{HCF} = 1$, $\text{LCM} = 2^3 \times 3^2 \times 5^2 = 1800$.

Q9. Can two numbers have 18 as their HCF?

Answer: The HCF of two numbers always divides both numbers. So **no** such pair exists.

3-Mark Questions

Q10. Prove that $\sqrt{7}$ is irrational.

Answer: Assume, to the contrary, that $\sqrt{7} = \frac{a}{b}$, where $a, b \in \mathbb{Z}$, $b \neq 0$. So $7b^2 = a^2$, and 7 divides a^2 . As 7 is prime, 7 divides a . Let $a = 7c$, then $7b^2 = 49c^2$, so $b^2 = 7c^2$. So 7 divides b^2 and hence b . This contradicts that a and b are coprime.

Q11. Given that $\sqrt{5}$ is irrational, prove that $2 - 3\sqrt{5}$ is irrational.

Answer: Assume, to the contrary, that $2 - 3\sqrt{5} = \frac{a}{b}$, where $a, b \in \mathbb{Z}$, $b \neq 0$. Then $3\sqrt{5} = 2 - \frac{a}{b} = \frac{2b - a}{b}$, so $\sqrt{5} = \frac{2b - a}{3b}$. The right-hand side is rational, so $\sqrt{5}$ would be rational. This contradicts the given that $\sqrt{5}$ is irrational.

Q12. Find the LCM and HCF of 336 and 54.

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Answer: $336 = 2^4 \times 3 \times 7$ and $54 = 2 \times 3^3$. $\text{HCF} = 2 \times 3 = 6$. $\text{LCM} = 2^4 \times 3^3 \times 7 = 16 \times 27 \times 7 = 3024$. $\text{LCM} \times \text{HCF} = 3024 \times 6 = 18144$. Product = $336 \times 54 = 18144$. Verified.

Q13. Three friends set off walking together. Their steps measure 40 cm, 42 cm and 45 cm. What is the minimum distance each should walk so that each covers it in a whole number of steps?

Answer: The distance must be a common multiple of all three step lengths, and the least one is the LCM. $40 = 2^3 \times 5$, $42 = 2 \times 3 \times 7$, $45 = 3^2 \times 5$. $\text{LCM} = 2^3 \times 3^2 \times 5 \times 7 = 2520 \text{ cm} = \mathbf{25 \text{ m } 20 \text{ cm}}$.

5-Mark Questions

Q14. Prove that $\sqrt{2}$ is irrational. Hence show that $3 + 2\sqrt{2}$ is irrational.

Answer: *Part 1.* Assume, to the contrary, that $\sqrt{2}$ is rational. Then $\sqrt{2} = a/b$, where a and b are coprime integers and $b \neq 0$. So $b\sqrt{2} = a$, and squaring, $2b^2 = a^2$. So 2 divides a^2 , and since 2 is prime, 2 divides a (Theorem 1.2). Write $a = 2m$, hence 2 divides b . So 2 is a common factor. Hence $\sqrt{2}$ is irrational.

Part 2. Assume, to the contrary, that $3 + 2\sqrt{2}$ is rational. Then $3 + 2\sqrt{2} = (p - 3q)/q$, so $\sqrt{2} = (p - 3q)/(2q)$. Since $\sqrt{2}$ is irrational, this would be rational. This contradicts Part 1.

Q15. (i) Find the HCF and LCM of 144, 180 and 192. Verify that the product of the three numbers is not equal to the product of their HCF and LCM. Verify your LCM using $\text{LCM}(p, q, r) = p \cdot q \cdot r / \text{HCF}(p, q, r)$.

Answer: (i) $144 = 2^4 \times 3^2$, $180 = 2^2 \times 3^2 \times 5$, $192 = 2^6 \times 3$.
 $\text{HCF} = 2^2 \times 3^2 = 36$.
 $\text{LCM} = 2^6 \times 3^2 \times 5 = 64 \times 45 = \mathbf{2880}$.

(ii) Product = $144 \times 180 \times 192 = 5080320$.
 $\text{HCF} \times \text{LCM} = 36 \times 2880 = 103680$.
These are unequal.

(iii) $\text{HCF}(144, 180) = 2^2 \times 3^2 = 36$, $\text{HCF}(180, 192) = 2^3 \times 3 = 24$, $\text{HCF}(144, 192) = 2^4 \times 3 = 48$.
Denominator = $36 \times 12 \times 48 = 20736$. Numerator = $144 \times 180 \times 192 = 5080320$.
 $5080320 \div 20736 = 2880$, which matches part (i).



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Common Mistakes and Examiner Tips

1. **Stopping a factor tree at a composite number.** Writing $156 = 4 \times 39$ is not a prime factorisation. Keep splitting until every factor is prime: $2^2 \times 3 \times 13$.
2. **Taking the greater power for HCF.** HCF uses the smallest power of each common prime; LCM uses the greatest power of every prime. Say "smallest, common" and "greatest, all" as you work.
3. **Putting a non-common prime into the HCF.** In $\text{HCF}(336, 54)$, the prime 7 appears only in 336, so it is left out of the HCF but included in the LCM.
4. **Using $\text{HCF} \times \text{LCM} = \text{product}$ for three numbers.** The rule is for two positive integers only. For 6, 72 and 120, $2160 \neq 51840$.
5. **Proving 4^n or 6^n cannot end in 0 by trying a few values of n.** Checking $n = 1, 2, 3$ proves nothing about all n. Write the factorisation, show 5 is absent and quote the uniqueness of the Fundamental Theorem of Arithmetic.
6. **Leaving out "a and b are coprime" in a factor is no contradiction.** State it on the conclusion.
7. **Forgetting " $b \neq 0$ ".** A rational number n assume a number equals a/b .
8. **Using Theorem 1.2 without saying the $a^2, 5$ divides a".** The theorem fails for composite numbers.
9. **Not naming the contradiction.** End with "contradicts that a and b are coprime" or "contradicts the conclusion".
10. **Claiming the answer "is rational" without proof.** a and b are integers, so the number is rational.
11. **Mixing up HCF and LCM in word problems.** "maximum number of columns" or "largest number of minutes, columns, cm."



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Quick Revision Points

- A prime has exactly two factors, 1 and itself. A composite has more than two factors.
- Fundamental Theorem of Arithmetic: every integer greater than 1 can be expressed as a product of primes, uniquely apart from the order of the factors.

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- $32760 = 2^3 \times 3^2 \times 5 \times 7 \times 13$.
- HCF = product of the smallest power of each common prime factor.
- LCM = product of the greatest power of each prime factor involved.
- $\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$ for two positive integers.
- For three numbers, $\text{HCF} \times \text{LCM}$ is generally not equal to the product.
- HCF always divides LCM. $\text{HCF} \leq \text{smallest number}$; $\text{LCM} \geq \text{largest number}$.
- A number ends in 0 only if its factorisation contains both 2 and 5. So 4^n , 6^n and 12^n never end in 0.
- Taking out a common factor shows a number is composite: $7 \times 11 \times 13 + 13 = 13 \times 78$.
- "Meet again" problems use LCM; "largest equal groups" problems use HCF.
- Theorem 1.2: if a prime p divides a^2 , then p divides a .
- Irrational: cannot be written as p/q with p, q integers and $q \neq 0$.
- Proof by contradiction: assume rational, show it is irrational.
- $\sqrt{p}$ is irrational for every prime p : $\sqrt{2}$, $\sqrt{3}$
- Rational $\pm$ irrational is irrational.
- Non-zero rational $\times$ irrational and non-zero irrational $\times$ irrational is irrational.
- Two irrationals can give a rational: $\sqrt{2} \times \sqrt{2} = 2$
- For $5 - \sqrt{3}$, $3\sqrt{2}$ and similar numbers, is rational.



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Weightage in Board Exams

Check the current CBSE course structure and how it is assessed in a very consistent way.

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Question type	What is usually asked
MCQ and Assertion-Reason	HCF or LCM of expressions like x^3y^2 and xy^3 ; a missing number from HCF and LCM; whether a number is rational or irrational; whether a power can end in 0
Short answers	Prime factorisation; HCF and LCM with verification of $\text{HCF} \times \text{LCM} = \text{product}$; showing a number is composite; 4^n or 6^n type reasoning
Proof questions	$\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ or $\sqrt{p}$ irrational; then a number such as $3 + 2\sqrt{5}$ or $5 - \sqrt{3}$ irrational
Case-based	Bells, traffic lights, runners or buses meeting again (LCM); columns, stacks or tiles (HCF)

In the irrationality proof, write every line: the assumption with “assuming that...”, the contradiction each use of Theorem 1.2 with “since p is p”, the conclusion with “we conclude that...”. Work through all of Exercises 1.1 and 1.2 and



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