



ChapterNotes

chapternotes.in · free NEET & JEE prep

CHAPTER NOTES

Quadratic Equations Class 10 Notes I CBSE Maths Chapter 4

- ✓ Chapter-wise notes & important questions
- ✓ Free gamified practice app — streaks, levels, revision
- ✓ Free mock & prediction papers with answer keys

Source: [https://chapternotes.in/cbse/class-10-maths/quadratic-equations-class-10-](https://chapternotes.in/cbse/class-10-maths/quadratic-equations-class-10-notes/)
notes/



Scan for the full chapter
page with practice
questions

Quadratic Equations Class 10 Notes | CBSE Maths

Chapter 4

Quadratic Equations is Chapter 4 of CBSE Class 10 Maths. It takes the quadratic polynomial $ax^2 + bx + c$ from Chapter 2, sets it equal to zero, and asks which values of x make the equation true. The chapter covers the standard form of a quadratic equation, turning word situations into equations, roots, solving by factorisation, the quadratic formula, and the discriminant that decides the nature of the roots.

For board exams this chapter sits in the Algebra unit. Its methods are fixed and every answer can be checked by substitution. Board papers regularly test it through a 1-mark item on the discriminant or on equal roots, a short factorisation or formula question, and a longer word problem on area, age, speed or work where you must form the equation, solve it and reject the root that makes no sense.

Quick Answer - Quadratic Equations in 30 Seconds

A quadratic equation is $ax^2 + bx + c = 0$ with a, b, c real and $a \neq 0$. It has at most two roots. Solve by **factorisation** (split the middle term, set each factor to zero) or by the **quadratic formula** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. The **discriminant** $D = b^2 - 4ac$ decides the nature of roots: $D > 0$ gives two distinct real roots, $D = 0$ gives two equal roots (each $-b/2a$), and $D < 0$ gives no real roots. In word problems, reject impossible roots.

All Key Formulas at a Glance

Idea	Formula or rule	Use it when
Standard form	$ax^2 + bx + c = 0, a \neq 0$	Reading a, b, c and checking if an equation is quadratic
Root of the equation	$ax^2 + bx + c = 0$	Verifying a root, or finding an unknown coefficient from a given root
Splitting the middle term	Two numbers with product $a \times c$ and sum b	Factorising $ax^2 + bx + c$
Zero product rule	$(px + q)(rx + s) = 0 \Rightarrow x = -q/p$ or $x = -s/r$	Finishing a factorisation
Quadratic formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Any quadratic with $b^2 - 4ac \geq 0$
Discriminant	$D = b^2 - 4ac$	Nature of roots without solving
Two distinct real roots	$D > 0$	"Real and unequal roots", "is it possible" checks
Two equal real roots	$D = 0$, each root $= -b/2a$	"Find k for equal roots"
No real roots	$D < 0$	Showing a situation is impossible

Key Concepts

1. What Is a Quadratic Equation

In Chapter 2 you met the **quadratic polynomial** $ax^2 + bx + c$, a polynomial of degree 2. When this polynomial is set equal to zero, the result is a **quadratic equation**.

A quadratic equation in the variable x is an equation of the form

$$ax^2 + bx + c = 0, \text{ where } a, b, c \text{ are real numbers and } a \neq 0$$

This arrangement, with the terms written in descending order of their degrees, is called the **standard form** of a quadratic equation. More generally, any equation $p(x) = 0$ where $p(x)$ is a polynomial of degree 2 is a quadratic equation.

- $2x^2 + x - 300 = 0$ is quadratic with $a = 2$, $b = 1$, $c = -300$.
- $4x - 3x^2 + 2 = 0$ is quadratic. In standard form it reads $-3x^2 + 4x + 2 = 0$, so $a = -3$, $b = 4$, $c = 2$.
- $x^2 - 9 = 0$ ($b = 0$) and $5x^2 + 2x = 0$ ($c = 0$) are also quadratic.

Why $a \neq 0$ matters. If $a = 0$, the x^2 term disappears and what is left, $bx + c = 0$, is a linear equation. The coefficients b and c are allowed to be zero; a is never allowed to be zero.

Simplify First, Then Decide

An equation may look quadratic and turn out linear, or look cubic and turn out quadratic. Expand everything, bring all terms to one side and look at the highest power of x that survives. In NCERT Example 2, $x(x + 1) + 8 = (x + 2)(x - 2)$ simplifies to $x + 12 = 0$ (linear, the x^2 terms cancel), while $(x + 2)^3 = x^3 - 4$ simplifies to $x^2 + 2x + 2 = 0$ (quadratic, the x^3 terms cancel). The rule: **the x^2 term must survive and higher powers must vanish**.

2. Representing Situations as Quadratic Equations

Many real problems about area, products of numbers, ages, cost and speed lead to quadratic equations. The skill tested here is translation: turning a paragraph into one equation in one unknown.

A four-step recipe

1. Choose one unknown and say clearly what it stands for, with units: "Let the breadth of the hall be x metres."
2. Write every other quantity in terms of x : length = $(2x + 1)$ m.
3. Use the one fact that has not been used yet to form the equation: area = 300 m^2 , so $x(2x + 1) = 300$.
4. Rearrange into standard form: $2x^2 + x - 300 = 0$.

This is the prayer hall problem from the start of the NCERT chapter: a hall of carpet area 300 m^2 whose length is one metre more than twice its breadth.

Standard Situation Types

Situation	Let	Equation formed
Two consecutive positive integers with product 306	smaller = x	$x(x + 1) = 306$, so $x^2 + x - 306 = 0$
Rohan's mother is 26 years older; product of their ages 3 years from now is 360	Rohan's age = x years	$(x + 3)(x + 29) = 360$, so $x^2 + 32x - 273 = 0$
Train covers 480 km; if speed were 8 km/h less it would take 3 hours more	speed = x km/h	$480/(x - 8) - 480/x = 3$, so $x^2 - 8x - 1280 = 0$
John and Jivanti have 45 marbles; each loses 5; product of what is left is 124	John's marbles = x	$(x - 5)(40 - x) = 124$, so $x^2 - 45x + 324 = 0$

The speed equation in detail. Time = distance/speed. At x km/h the train takes $480/x$ hours; at $(x - 8)$ km/h it takes $480/(x - 8)$ hours, which is 3 hours more. So $480/(x - 8) - 480/x = 3$. Taking the LCM on the left, $480[x - (x - 8)] / [x(x - 8)] = 3$, that is $3840 = 3x(x - 8)$, or $3x^2 - 24x - 3840 = 0$. Dividing by 3 gives $x^2 - 8x - 1280 = 0$.

When a question says only "represent the situation", stop at the standard form. When it says "find", carry on and solve.

3. Roots of a Quadratic Equation

A real number α is called a **root** of the quadratic equation $ax^2 + bx + c = 0$ ($a \neq 0$) if

$$a\alpha^2 + b\alpha + c = 0$$

We also say that $x = \alpha$ is a **solution** of the equation, or that α **satisfies** the equation. For example, putting $x = 1$ in $2x^2 - 3x + 1$ gives $2 - 3 + 1 = 0$, so 1 is a root of $2x^2 - 3x + 1 = 0$.

Roots and zeroes are the same numbers. The zeroes of the quadratic polynomial $ax^2 + bx + c$ and the roots of the quadratic equation $ax^2 + bx + c = 0$ are identical. Saying "1 is a root of $2x^2 - 3x + 1 = 0$ " and "1 is a zero of $2x^2 - 3x + 1$ " means the same thing.

At most two roots. A quadratic polynomial has at most two zeroes (Chapter 2), so a quadratic equation has at most two roots. It can have two different real roots, one real root repeated twice, or no real roots at all.

Using a Given Root

A very common short question gives one root and an unknown coefficient. Substitute the root and solve for the coefficient.

Illustration: If $x = 2$ is a root of $x^2 + kx - 6 = 0$, find k and the other root.

Put $x = 2$: $4 + 2k - 6 = 0$, so $2k = 2$ and $k = 1$. The equation is $x^2 + x - 6 = 0 = (x + 3)(x - 2)$, so the other root is -3 .

4. Solving by Factorisation

If the quadratic $ax^2 + bx + c$ can be written as a product of two linear factors, the roots come from setting each factor equal to zero. This works because of the **zero product rule**: if the product of two numbers is 0, at least one of them must be 0.

Splitting the Middle Term

You learnt this in Class 9. To factorise $ax^2 + bx + c$:

1. Multiply a and c .
2. Find two numbers whose **product is $a \times c$** and whose **sum is b** .
3. Rewrite bx as the sum of two terms using those numbers.
4. Group in pairs and take out the common factor from each pair.
5. Take out the common bracket to get two linear factors.

Illustration (NCERT Example 3): $2x^2 - 5x + 3 = 0$.

$a \times c = 2 \times 3 = 6$ and $b = -5$. The pair -2 and -3 has product 6 and sum -5 .

$$2x^2 - 2x - 3x + 3 = 2x(x - 1) - 3(x - 1) = (2x - 3)(x - 1)$$

So $2x - 3 = 0$ or $x - 1 = 0$, giving **$x = 3/2$ or $x = 1$** .

Repeated Roots

Sometimes both factors are the same. NCERT Example 5 solves $3x^2 - 2\sqrt{6}x + 2 = 0$. Here $a \times c = 6$, and the split is $-\sqrt{6}x - \sqrt{6}x$:

$$3x^2 - \sqrt{6}x - \sqrt{6}x + 2 = \sqrt{3}x(\sqrt{3}x - \sqrt{2}) - \sqrt{2}(\sqrt{3}x - \sqrt{2}) = (\sqrt{3}x - \sqrt{2})(\sqrt{3}x - \sqrt{2})$$

The factor $\sqrt{3}x - \sqrt{2} = 0$ gives $x = \sqrt{2}/\sqrt{3} = \sqrt{2/3}$. The root is **repeated twice**, once for each factor, so the roots are $\sqrt{2/3}, \sqrt{2/3}$.

A perfect square such as $100x^2 - 20x + 1 = (10x - 1)^2$ behaves the same way: roots $1/10, 1/10$.

Surds and fractions. For $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$, $a \times c = 10$ and $5 + 2 = 7$, so split $7x$ as $5x + 2x$ and write $2x$ as $\sqrt{2} \times \sqrt{2}x$. With fractions, multiply through first: $2x^2 - x + 1/8 = 0$ becomes $16x^2 - 8x + 1 = (4x - 1)^2 = 0$.

Never divide both sides by x . In $x^2 = 5x$, dividing by x throws away the root $x = 0$. Write $x^2 - 5x = 0$, so $x(x - 5) = 0$, giving $x = 0$ or $x = 5$.

5. The Quadratic Formula

Factorisation is fast when the numbers are friendly. The quadratic formula works for every quadratic equation that has real roots. The roots of $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \text{ provided } b^2 - 4ac \geq 0$$

The $\pm$ sign gives two roots: $\frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $\frac{-b - \sqrt{b^2 - 4ac}}{2a}$.

History from NCERT: Brahmagupta (C.E. 598 to 665) gave a formula for $ax^2 + bx = c$, and Sridharacharya (C.E. 1025) derived the quadratic formula by completing the square, as quoted by Bhaskara II.

Where the Formula Comes From

The NCERT chapter states the formula and asks you to apply it. The working below shows why it is true, using Sridharacharya's idea of completing the square.

1. Divide $ax^2 + bx + c = 0$ by a : $x^2 + (b/a)x + c/a = 0$.
2. Add and subtract $(b/2a)^2$: $[x^2 + (b/a)x + (b/2a)^2] - (b/2a)^2 + c/a = 0$.
3. The bracket is a perfect square: $(x + b/2a)^2 = b^2/4a^2 - c/a = (b^2 - 4ac) / 4a^2$.
4. If $b^2 - 4ac \geq 0$, take square roots: $x + b/2a = \pm \sqrt{(b^2 - 4ac) / 4a^2}$.
5. So $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Step 3 also shows why the formula needs $b^2 - 4ac \geq 0$: the left side is a square, which can never be negative.

Using the Formula Without Slips

1. Write the equation in standard form. Everything must be on one side, equal to 0.
2. Read a , b and c **with their signs**. In $x^2 + 7x - 60 = 0$, $c = -60$, with its minus sign.
3. Work out $b^2 - 4ac$ on its own line first.
4. Simplify the square root ($\sqrt{12} = 2\sqrt{3}$, $\sqrt{289} = 17$).
5. Divide the **whole** numerator by $2a$, then simplify each root.

Illustration (from NCERT Example 8): $x^2 + 7x - 60 = 0$. Here $a = 1$, $b = 7$, $c = -60$.

$$b^2 - 4ac = 49 - 4(1)(-60) = 49 + 240 = 289, \text{ and } \sqrt{289} = 17.$$

$$x = \frac{-7 \pm 17}{2}, \text{ so } x = 10/2 = \mathbf{5} \text{ or } x = -24/2 = \mathbf{-12}.$$

Illustration with a surd answer: $2x^2 - 6x + 3 = 0$. Here $a = 2$, $b = -6$, $c = 3$.

$$b^2 - 4ac = 36 - 24 = 12, \text{ and } \sqrt{12} = 2\sqrt{3}.$$

$x = \frac{(6 \pm 2\sqrt{3})}{4} = \mathbf{(3 \pm \sqrt{3}) / 2}$. This equation cannot be factorised over whole numbers, which is exactly when the formula earns its place.

Choosing a method: if the question names a method, use it, since the method is marked. Otherwise factorise when a split is quick, and use the formula for large constants (such as -1280) or when no split appears within about 30 seconds.

6. Discriminant and Nature of Roots

The expression under the square root in the quadratic formula decides everything about the roots. It is called the **discriminant**:

$$D = b^2 - 4ac$$

It “discriminates” between the three possible kinds of roots.

Value of D	Nature of roots	The roots
$D > 0$	Two distinct real roots	$(-b + \sqrt{D})/2a$ and $(-b - \sqrt{D})/2a$
$D = 0$	Two equal real roots (coincident roots)	$-b/2a$ and $-b/2a$
$D < 0$	No real roots	No real number has a negative square

When $D = 0$, the formula gives $x = (-b \pm 0)/2a$, so both roots equal $-b/2a$. When $D < 0$, $\sqrt{D}$ is the square root of a negative number, and no real number squares to give a negative result, so the equation has no real roots.

Useful extra: with rational a, b, c , a perfect-square D (such as 289) gives rational roots; a positive non-square D (such as 12) gives surd roots.

Worked Illustrations from NCERT

$3x^2 - 2x + 1/3 = 0$: $D = (-2)^2 - 4 \times 3 \times (1/3) = 4 - 4 = 0$. Two equal real roots, each $-b/2a = 2/6 = 1/3$.

Finding an Unknown for Equal Roots

“Find k so that the equation has two equal roots” means: write D in terms of k , set $D = 0$, and solve.

$2x^2 + kx + 3 = 0$: $D = k^2 - 4 \times 2 \times 3 = k^2 - 24$. Setting $D = 0$ gives $k^2 = 24$, so $k = \pm 2\sqrt{6}$.

For “real roots” the condition is $D \geq 0$. For “real and distinct roots” it is $D > 0$. For “no real roots” it is $D < 0$. Read the wording carefully before writing the condition.

Is the Situation Possible

The discriminant also answers practical questions. Form the equation from the situation. If $D \geq 0$ the situation is possible and you go on to find the values; if $D < 0$ it is impossible.

NCERT Example 8, the pole in a circular park. A park is a circle of diameter 13 m, with two gates A and B at opposite ends of a diameter. A pole P is to stand on the boundary so that the difference of its distances from the two gates is 7 m. Let $BP = x$ m, so $AP = (x + 7)$ m. Since AB is a diameter, the angle APB in the semicircle is 90° , so by Pythagoras theorem

$(x + 7)^2 + x^2 = 13^2$, which gives $2x^2 + 14x + 49 - 169 = 0$, that is $x^2 + 7x - 60 = 0$.

$D = 49 + 240 = 289 > 0$, so the pole can be placed. The roots are 5 and -12 (worked in Concept 5). A distance cannot be negative, so $x = 5$. The pole stands **5 m from gate B and 12 m from gate A**. Check: $5^2 + 12^2 = 25 + 144 = 169 = 13^2$.

7. Solving Word Problems and Rejecting Roots

The equation gives two roots; the situation usually accepts only one. After solving, test each root against the story.

- **Lengths, ages, speeds, times, counts:** must be positive. Reject negative roots.
- **Counts of objects or people:** must be whole numbers. Reject fractions.
- **A quantity used in a subtraction:** check that every derived quantity is positive too. If a tap takes x hours and the other takes $(x - 10)$ hours, $x = 3.75$ gives a negative time for the second tap, so it is rejected even though 3.75 itself is positive.
- **Both roots may be valid.** The marbles equation $x^2 - 45x + 324 = (x - 9)(x - 36) = 0$ gives 36 and 9: John has 36 and Jivanti 9, or the other way round. The toys equation gives 25 or 30 toys, and both cost Rs 750 in total. State both when both fit.

Relations to keep ready: time = distance/speed; upstream speed = $u - v$ and downstream speed = $u + v$ for a boat of speed u in a stream of speed v ; a job done alone in x hours means $1/x$ of it is done in one hour; ages n years ago are $x - n$; consecutive integers are x and $x + 1$.

Always finish with a check and a sentence. Substitute the answer back into the original story as well as the equation, and write the answer with units: "The speed of the train is 40 km/h."

Formula and Theorem Sheet

Result	Statement	When to use it
Standard form	$ax^2 + bx + c = 0$, a, b, c real, $a \neq 0$	Before reading a, b, c or applying any formula
Root	α is a root if $a\alpha^2 + b\alpha + c = 0$	Verifying roots; finding k from a given root
Splitting the middle term	Numbers p, q with $p + q = b$ and $pq = ac$	Factorising $ax^2 + bx + c$
Quadratic formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, when $b^2 - 4ac \geq 0$	Any quadratic with real roots
Discriminant	$D = b^2 - 4ac$	Nature of roots without solving
$D > 0$	Two distinct real roots	"Real and unequal", "possible"
$D = 0$	Two equal real roots, each $-b/2a$	"Equal roots", "coincident roots"
$D < 0$	No real roots	"Not possible" situations

Important Definitions

Term	Definition
Quadratic equation	An equation of the form $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$; equivalently $p(x) = 0$ where $p(x)$ has degree 2
Standard form	$ax^2 + bx + c = 0$, $a \neq 0$, with the terms written in descending order of their degrees
Root (solution)	A real number α such that $a\alpha^2 + b\alpha + c = 0$; we say α satisfies the equation
Quadratic formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, giving the roots of $ax^2 + bx + c = 0$ when $b^2 - 4ac \geq 0$
Discriminant	The expression $b^2 - 4ac$, which determines whether $ax^2 + bx + c = 0$ has real roots
Distinct real roots	Two different real roots; occur when $b^2 - 4ac > 0$
Equal (coincident) roots	Two real roots with the same value $-b/2a$; occur when $b^2 - 4ac = 0$
No real roots	The case $b^2 - 4ac < 0$, when no real number satisfies the equation

Solved Examples (NCERT-Based)

Example 1: Check whether these are quadratic equations (NCERT Exercise 4.1)

(i) $(x - 2)(x + 1) = (x - 1)(x + 3)$ (ii) $(x + 2)^3 = 2x(x^2 - 1)$ (iii) $x^3 - 4x^2 - x + 1 = (x - 2)^3$

Solution:

(i) LHS = $x^2 - x - 2$; RHS = $x^2 + 2x - 3$. So $x^2 - x - 2 = x^2 + 2x - 3$, giving $-3x + 1 = 0$. The x^2 terms cancel. **Not quadratic.**

(ii) LHS = $x^3 + 6x^2 + 12x + 8$; RHS = $2x^3 - 2x$. So $x^3 - 6x^2 - 14x - 8 = 0$, which has degree 3. **Not quadratic.**

(iii) LHS = $x^3 - 4x^2 - x + 1$; RHS = $x^3 - 6x^2 + 12x - 8$. So $x^3 - 4x^2 - x + 1 = x^3 - 6x^2 + 12x - 8$, giving $2x^2 - 13x + 9 = 0$. The x^3 terms cancel. **Quadratic.**

Example 2: Rohan's age (NCERT Exercise 4.1)

Rohan's mother is 26 years older than him. The product of their ages 3 years from now will be 360. Find Rohan's present age.

Solution: Let Rohan's age be x years; his mother's age is $x + 26$. In 3 years: $x + 3$ and $x + 29$.

$$(x + 3)(x + 29) = 360 \Rightarrow x^2 + 32x + 87 = 360 \Rightarrow x^2 + 32x - 273 = 0.$$

Product -273 , sum 32 : the pair 39 and -7 . So $(x + 39)(x - 7) = 0$, giving $x = 7$ or $x = -39$.

Age cannot be negative, so $x = 7$. Check: in 3 years the ages are 10 and 36, and $10 \times 36 = 360$. **Rohan is 7 years old.**

Example 3: Roots by factorisation (NCERT Exercise 4.2)

Find the roots of (i) $x^2 - 3x - 10 = 0$ and (ii) $2x^2 + x - 6 = 0$.

Solution:

(i) Product -10 , sum -3 : -5 and 2 . $x^2 - 5x + 2x - 10 = x(x - 5) + 2(x - 5) = (x - 5)(x + 2)$. **Roots: 5 and -2 .**

(ii) $a \times c = -12$, sum 1 : 4 and -3 . $2x^2 + 4x - 3x - 6 = 2x(x + 2) - 3(x + 2) = (2x - 3)(x + 2)$. **Roots: $3/2$ and -2 .**

Example 4: Equal roots from a perfect square (NCERT Exercise 4.2)

Solve (i) $2x^2 - x + 1/8 = 0$ and (ii) $100x^2 - 20x + 1 = 0$.

Solution:

(i) Multiply by 8: $16x^2 - 8x + 1 = 0$. Product 16 , sum -8 : -4 and -4 . $16x^2 - 4x - 4x + 1 = 4x(4x - 1) - 1(4x - 1) = (4x - 1)^2$. **Roots: $1/4$, $1/4$.**

(ii) Product 100 , sum -20 : -10 and -10 . $100x^2 - 10x - 10x + 1 = 10x(10x - 1) - 1(10x - 1) = (10x - 1)^2$. **Roots: $1/10$, $1/10$.**

Example 5: The right triangle (NCERT Exercise 4.2)

The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

Solution: Let the base be x cm; the altitude is $(x - 7)$ cm. By Pythagoras theorem:

$$x^2 + (x - 7)^2 = 13^2 \Rightarrow x^2 + x^2 - 14x + 49 = 169 \Rightarrow 2x^2 - 14x - 120 = 0 \Rightarrow x^2 - 7x - 60 = 0.$$

Product -60 , sum -7 : -12 and 5 . $(x - 12)(x + 5) = 0$, so $x = 12$ or $x = -5$.

A side cannot be negative, so $x = 12$. **Base 12 cm, altitude 5 cm.** Check: $12^2 + 5^2 = 144 + 25 = 169 = 13^2$.

Example 6: Pottery articles (NCERT Exercise 4.2)

On a particular day the cost of production of each pottery article (in rupees) was 3 more than twice the number of articles produced. The total cost that day was Rs 90. Find the number of articles and the cost of each.

Solution: Let x articles be produced; cost of each = $(2x + 3)$ rupees.

$$x(2x + 3) = 90 \Rightarrow 2x^2 + 3x - 90 = 0. \text{ Here } a = 2, b = 3, c = -90.$$

$$D = 3^2 - 4(2)(-90) = 9 + 720 = 729, \text{ and } \sqrt{729} = 27.$$

$$x = \frac{-3 \pm 27}{4}, \text{ so } x = \frac{24}{4} = 6 \text{ or } x = \frac{-30}{4} = -7.5.$$

The number of articles must be a positive whole number, so $x = 6$. Cost of each = $2(6) + 3 = 15$. **6 articles at Rs 15 each** (check: $6 \times 15 = 90$).

Example 7: Nature of roots, and roots if real (NCERT Exercise 4.3)

(i) $2x^2 - 3x + 5 = 0$ (ii) $3x^2 - 4\sqrt{3}x + 4 = 0$ (iii) $2x^2 - 6x + 3 = 0$

Solution:

(i) $D = (-3)^2 - 4(2)(5) = 9 - 40 = -31 < 0$. **No real roots.**

(ii) $D = (-4\sqrt{3})^2 - 4(3)(4) = 48 - 48 = 0$. **Two equal real roots**, each $-b/2a = 4\sqrt{3}/6 = 2\sqrt{3}/3 = 2/\sqrt{3}$.

(iii) $D = (-6)^2 - 4(2)(3) = 36 - 24 = 12 > 0$. **Two distinct real roots:** $x = (6 \pm \sqrt{12})/4 = (6 \pm 2\sqrt{3})/4 = (3 \pm \sqrt{3})/2$.

Example 8: Value of k for equal roots (NCERT Exercise 4.3)

Find k so that (i) $2x^2 + kx + 3 = 0$ and (ii) $kx(x - 2) + 6 = 0$ have two equal roots.

Solution:

(i) $a = 2$, $b = k$, $c = 3$. $D = k^2 - 24 = 0$, so $k^2 = 24$ and **$k = \pm 2\sqrt{6}$** .

(ii) Expand: $kx^2 - 2kx + 6 = 0$, so $a = k$, $b = -2k$, $c = 6$. $D = 4k^2 - 24k = 4k(k - 6) = 0$, so $k = 0$ or $k = 6$. If $k = 0$ the equation becomes $6 = 0$, which is not a quadratic equation at all ($a = k$ must be non-zero). So **$k = 6$** . Check: $6x^2 - 12x + 6 = 6(x - 1)^2$, equal roots 1, 1.

Example 9: The friends' ages, an impossible situation (NCERT Exercise 4.3)

The sum of the ages of two friends is 20 years. Four years ago, the product of their ages was 48. Is this possible? If so, find their present ages.

Solution: Let one friend be x years old; the other is $20 - x$. Four years ago: $x - 4$ and $16 - x$.

$$(x - 4)(16 - x) = 48 \Rightarrow 16x - x^2 - 64 + 4x = 48 \Rightarrow -x^2 + 20x - 112 = 0 \Rightarrow x^2 - 20x + 112 = 0.$$

$$D = (-20)^2 - 4(1)(112) = 400 - 448 = -48 < 0.$$

No real roots, so **the situation is not possible**.

Example 10: The rectangular park (NCERT Exercise 4.3)

Is it possible to design a rectangular park of perimeter 80 m and area 400 m^2 ? If so, find its length and breadth.

Solution: Length + breadth = $80/2 = 40$. Let the length be x m; breadth = $(40 - x)$ m. $x(40 - x) = 400 \Rightarrow x^2 - 40x + 400 = 0$.

$D = 1600 - 1600 = 0$, so it is possible, with equal roots $x = 40/2 = 20$. **Length 20 m, breadth 20 m:** the park is a square. (For the mango grove in the same exercise, length twice breadth and area 800 m^2 , $2x^2 = 800$ gives 20 m by 40 m.)

Example 11: Speed of the train (NCERT Exercise 4.1, solved)

A train travels 480 km at a uniform speed. If the speed had been 8 km/h less, it would have taken 3 hours more. Find the speed of the train.

Solution: Let the speed be x km/h. From Concept 2, $x^2 - 8x - 1280 = 0$.

$$D = 64 + 5120 = 5184, \text{ and } \sqrt{5184} = 72.$$

$$x = (8 \pm 72)/2, \text{ so } x = 40 \text{ or } x = -32. \text{ Speed cannot be negative, so } x = 40.$$

Check: $480/40 = 12$ hours; $480/32 = 15$ hours; the difference is 3 hours. **The speed of the train is 40 km/h.**

Example 12: Two number problems (NCERT Exercise 4.2)

(i) Find two numbers whose sum is 27 and product is 182. (ii) Find two consecutive positive integers, the sum of whose squares is 365.

Solution:

(i) Let one number be x ; the other is $27 - x$. Then $x(27 - x) = 182 \Rightarrow 27x - x^2 = 182 \Rightarrow x^2 - 27x + 182 = 0$. Product 182, sum -27 : -13 and -14 . So $(x - 13)(x - 14) = 0$, giving $x = 13$ or $x = 14$. Either way the pair is **13 and 14** (check: $13 + 14 = 27$, $13 \times 14 = 182$).

(ii) Let the integers be x and $x + 1$. Then $x^2 + (x + 1)^2 = 365 \Rightarrow 2x^2 + 2x + 1 = 365 \Rightarrow 2x^2 + 2x - 364 = 0 \Rightarrow x^2 + x - 182 = 0$. Product -182 , sum 1: 14 and -13 . So $(x + 14)(x - 13) = 0$, giving $x = 13$ or $x = -14$. The integers must be positive, so $x = 13$. **The integers are 13 and 14** (check: $169 + 196 = 365$).

Competency-Based Questions (with answers)

1. Case-based: The school garden

A school is laying out a rectangular vegetable garden. The length is 5 m more than the breadth and the area is 84 m^2 .

(a) Form the quadratic equation for the breadth x . $x(x + 5) = 84$, so $x^2 + 5x - 84 = 0$.

(b) Find the length and breadth. Product -84 , sum 5: 12 and -7 . $(x + 12)(x - 7) = 0$, so $x = 7$ (rejecting -12). Breadth 7 m, length 12 m.

(c) How much fencing is needed to go round the garden once? Perimeter $= 2(12 + 7) = 38$ m.

2. Case-based: How high does the ball go

A ball is thrown straight up. Its height above the ground after t seconds is $h = 20t - 5t^2$ metres.

(a) At what times is the ball 15 m high? $20t - 5t^2 = 15 \Rightarrow 5t^2 - 20t + 15 = 0 \Rightarrow t^2 - 4t + 3 = 0 \Rightarrow (t - 1)(t - 3) = 0$. At $t = 1$ s on the way up and $t = 3$ s on the way down.

(b) Can the ball reach 25 m? $5t^2 - 20t + 25 = 0 \Rightarrow t^2 - 4t + 5 = 0$. $D = 16 - 20 = -4 < 0$. No real time exists, so the ball never reaches 25 m.

(c) What is the greatest height? For a height of h metres, $5t^2 - 20t + h = 0$ has $D = 400 - 20h$, which is ≥ 0 only when $h \leq 20$. So the ball cannot rise above 20 m. At $h = 20$, $t^2 - 4t + 4 = 0$ has $D = 0$ and the repeated root $t = 2$. The greatest height is 20 m, reached at $t = 2$ s.

3. Case-based: The motorboat and the stream

A motorboat whose speed in still water is 18 km/h takes 1 hour more to go 24 km upstream than to return downstream to the same spot.

(a) Write the upstream and downstream speeds if the stream flows at x km/h. Upstream $(18 - x)$ km/h; downstream $(18 + x)$ km/h.

(b) Form the equation. $24/(18 - x) - 24/(18 + x) = 1$. Taking the LCM: $24[(18 + x) - (18 - x)] = (18 - x)(18 + x)$, so $48x = 324 - x^2$, that is $x^2 + 48x - 324 = 0$.

(c) Find the speed of the stream. $D = 2304 + 1296 = 3600$, $\sqrt{D} = 60$. $x = (-48 \pm 60)/2 = 6$ or -54 . Speed is positive, so the stream flows at **6 km/h**. Check: $24/12 = 2$ h upstream, $24/24 = 1$ h downstream, a difference of 1 hour.

4. Source-based: The Babylonian problem

The NCERT chapter notes that the Babylonians knew how to find two positive numbers with a given positive sum and a given positive product, and that this is the same as solving $x^2 - px + q = 0$, where p is the sum and q is the product.

(a) Find two numbers with sum 13 and product 40. $x^2 - 13x + 40 = 0 \Rightarrow (x - 5)(x - 8) = 0$. The numbers are 5 and 8.

(b) For a sum of 10, what is the largest possible product? $x^2 - 10x + q = 0$ has real roots only if $D = 100 - 4q \geq 0$, that is $q \leq 25$. The largest product is 25, when both numbers are 5.

5. Assertion-Reason

Assertion (A): The equation $x^2 + 4x + 5 = 0$ has no real roots.

Reason (R): A quadratic equation $ax^2 + bx + c = 0$ has no real roots when $b^2 - 4ac < 0$.

Choose: (a) Both A and R are true and R is the correct explanation of A. (b) Both A and R are true but R is not the correct explanation of A. (c) A is true but R is false. (d) A is false but R is true.

Answer: (a) Both A and R are true, and R is the correct explanation of A. $D = 16 - 20 = -4 < 0$.

6. Assertion-Reason

Assertion (A): $(x - 1)^2 = x^2 - 5$ is not a quadratic equation.

Reason (R): An equation is quadratic whenever an x^2 term appears in it as written.

Answer: (c) A is true but R is false. Expanding gives $x^2 - 2x + 1 = x^2 - 5$, so $-2x + 6 = 0$, which is linear. The test is applied after simplifying, so R is false.

7. Error analysis: The lost root

A student solves $3x^2 = 12x$ like this: "Divide both sides by $3x$, so $x = 4$." Identify the error and give the complete answer.

Answer: Dividing by $3x$ assumes $x \neq 0$, which throws away the root $x = 0$. Correct method: $3x^2 - 12x = 0 \Rightarrow 3x(x - 4) = 0 \Rightarrow x = 0$ or $x = 4$. A quadratic can have two roots, so both must be found.

8. Competency MCQ: Which equation has two distinct real roots

(a) $x^2 + x + 1 = 0$ (b) $2x^2 - 4x + 2 = 0$ (c) $x^2 - 3x - 4 = 0$ (d) $4x^2 + 4x + 5 = 0$

Answer: (c). D values: (a) $1 - 4 = -3$; (b) $16 - 16 = 0$; (c) $9 + 16 = 25$; (d) $16 - 80 = -64$. Only (c) has $D > 0$; its roots are 4 and -1 . Shortcut: when a and c have opposite signs, $-4ac$ is positive, so D is positive and the roots are always real and distinct.

Important Questions for Board Exams

1-Mark Questions

- Find the discriminant of $2x^2 - 4x + 3 = 0$. $D = (-4)^2 - 4(2)(3) = 16 - 24 = -8$.
- If $x = 2$ is a root of $3x^2 + kx - 2 = 0$, find k . $12 + 2k - 2 = 0$, so $k = -5$.
- Find the values of k for which $x^2 + kx + 9 = 0$ has equal roots. $k^2 - 36 = 0$, so $k = \pm 6$.
- State the nature of the roots of $x^2 - 4x + 4 = 0$. $D = 16 - 16 = 0$, so two equal real roots, each 2.

2-Mark Questions

- Find the roots of $6x^2 - x - 2 = 0$ by factorisation. $a \times c = -12$, sum -1 : 3 and -4 . $6x^2 + 3x - 4x - 2 = 3x(2x + 1) - 2(2x + 1) = (3x - 2)(2x + 1)$. Roots $2/3$ and $-1/2$.
- Solve $\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$. $a \times c = 21$, sum 10: 7 and 3, with $3x = \sqrt{3} \times \sqrt{3}x$. $\sqrt{3}x^2 + 3x + 7x + 7\sqrt{3} = \sqrt{3}x(x + \sqrt{3}) + 7(x + \sqrt{3}) = (x + \sqrt{3})(\sqrt{3}x + 7)$. Roots $-\sqrt{3}$ and $-7/\sqrt{3} = -7\sqrt{3}/3$.
- Solve $x^2 - 2ax + (a^2 - b^2) = 0$. The split needs two numbers with product $a^2 - b^2 = (a + b)(a - b)$ and sum $-2a$: these are $-(a + b)$ and $-(a - b)$. So $[x - (a + b)][x - (a - b)] = 0$, and the roots are $a + b$ and $a - b$.

3-Mark Questions

- Solve $2x^2 + x - 4 = 0$ using the quadratic formula. $a = 2$, $b = 1$, $c = -4$. $D = 1 + 32 = 33 > 0$. $x = (-1 \pm \sqrt{33})/4$. The roots are $(-1 + \sqrt{33})/4$ and $(-1 - \sqrt{33})/4$.
- Find p for which $(p + 4)x^2 + (p + 1)x + 1 = 0$ has equal roots, and find the roots. $D = (p + 1)^2 - 4(p + 4) = p^2 - 2p - 15 = (p - 5)(p + 3) = 0$, so $p = 5$ or $p = -3$ (both keep $p + 4 \neq 0$). For $p = 5$: $9x^2 + 6x + 1 = (3x + 1)^2$, root $-1/3$ (twice). For $p = -3$: $x^2 - 2x + 1 = (x - 1)^2$, root 1 (twice).
- The product of Sunita's age (in years) two years ago and her age four years from now is one more than twice her present age. Find her present age. Let her age be x . $(x - 2)(x + 4) = 2x + 1 \Rightarrow x^2 + 2x - 8 = 2x + 1 \Rightarrow x^2 = 9 \Rightarrow x = 3$ (rejecting -3). Sunita is 3 years old. Check: $1 \times 7 = 7 = 2(3) + 1$.

5-Mark Questions

- 1. An express train takes 1 hour less than a passenger train to travel 132 km between Mysuru and Bengaluru, ignoring the time they stop at intermediate stations. The average speed of the express train is 11 km/h more than that of the passenger train. Find the average speed of each train.** Let the passenger train's speed be x km/h; the express train's speed is $(x + 11)$ km/h. Time difference: $132/x - 132/(x + 11) = 1$. LCM: $132 \times 11 = x(x + 11)$, so $x^2 + 11x - 1452 = 0$. $D = 121 + 5808 = 5929 = 77^2$. $x = (-11 \pm 77)/2 = 33$ or -44 . Speed is positive, so the passenger train runs at **33 km/h** and the express at **44 km/h**. Check: $132/33 = 4$ h, $132/44 = 3$ h.
 - 2. Two water taps together can fill a tank in $75/8$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank alone. Find the time each tap takes on its own.** Let the smaller tap take x hours; the larger takes $(x - 10)$ hours. In one hour they fill $1/x + 1/(x - 10) = 8/75$ of the tank. So $75(2x - 10) = 8x(x - 10) \Rightarrow 150x - 750 = 8x^2 - 80x \Rightarrow 8x^2 - 230x + 750 = 0 \Rightarrow 4x^2 - 115x + 375 = 0$. $D = 13225 - 6000 = 7225 = 85^2$. $x = (115 \pm 85)/8 = 25$ or 3.75 . If $x = 3.75$, the larger tap would take -6.25 hours, which is impossible. So the **smaller tap takes 25 hours and the larger 15 hours**. Check: $1/25 + 1/15 = 3/75 + 5/75 = 8/75$.
-

Common Mistakes and Examiner Tips

- 1. Deciding "quadratic or not" before simplifying.** $x(x + 1) + 8 = (x + 2)(x - 2)$ looks quadratic and is linear; $(x + 2)^3 = x^3 - 4$ looks cubic and is quadratic. Expand, collect everything on one side, then look at the highest power.
 - 2. Forgetting $a \neq 0$ in "find k " questions.** In $kx(x - 2) + 6 = 0$, setting $D = 0$ gives $k = 0$ or 6 . $k = 0$ kills the x^2 term, so the answer is $k = 6$ only. Always test each value of k against the leading coefficient.
 - 3. Dropping the sign of b or c .** In $2x^2 - 6x + 3 = 0$, $b = -6$ and $-b = 6$. In $x^2 + 7x - 60 = 0$, $c = -60$ and $-4ac = +240$. Write " $a =$, $b =$, $c =$ " with signs before substituting.
 - 4. Dividing only part of the numerator by $2a$.** $x = -b \pm \sqrt{D} / 2a$ is wrong; the whole of $(-b \pm \sqrt{D})$ is divided by $2a$. Use brackets.
 - 5. Cancelling x from both sides.** Dividing $x^2 = 5x$ by x loses the root $x = 0$. Bring everything to one side and factorise.
 - 6. Setting each factor equal to the right-hand number.** From $(x - 3)(x + 2) = 6$ you cannot write $x - 3 = 6$. The zero product rule works only when the right side is 0. Expand first: $x^2 - x - 12 = 0 = (x - 4)(x + 3)$, roots 4 and -3 .
 - 7. Keeping a root that the situation rejects.** Negative lengths, ages, speeds, times, and fractional counts must be struck out, with a reason written: " $x = -12$ is rejected as distance cannot be negative." Writing the reason shows the examiner why the root was dropped.
 - 8. Writing the wrong condition for the words used.** "Equal roots": $D = 0$. "Real roots": $D \geq 0$. "Distinct real roots": $D > 0$. "No real roots": $D < 0$. Mixing $\geq$ and $>$ changes the answer to "find the range of k " questions.
 - 9. Using the formula when the question says "by factorisation".** The method is part of what is marked, so follow the method the question names.
 - 10. Stopping at " $x = 40$ ".** Finish with units and a sentence: "The speed of the train is 40 km/h."
 - 11. Subtracting times the wrong way round.** The slower journey takes longer: write (longer time) $-$ (shorter time) = difference.
-

Quick Revision Points

- Quadratic equation: $ax^2 + bx + c = 0$, with a, b, c real and $a \neq 0$
- b or c can be 0; a can never be 0
- Decide whether an equation is quadratic only after expanding and simplifying
- α is a root if $a\alpha^2 + b\alpha + c = 0$
- Roots of the equation are the zeroes of the polynomial; at most two roots
- Given a root, substitute it to find an unknown coefficient
- Factorisation: split b into two numbers with product ac and sum b
- Zero product rule needs 0 on the right side first
- A repeated factor gives a repeated root: $(10x - 1)^2 = 0$ gives $1/10, 1/10$
- Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, valid when $b^2 - 4ac \geq 0$
- The formula comes from completing the square (Sridharacharya)
- Discriminant $D = b^2 - 4ac$
- $D > 0$: two distinct real roots; $D = 0$: two equal roots, each $-b/2a$; $D < 0$: no real roots
- If a and c have opposite signs, $D > 0$ automatically
- Equal roots: set $D = 0$, then reject any k that makes $a = 0$
- "Is it possible": form the equation and check $D \geq 0$
- Word problems: define x with units, solve, reject impossible roots with a reason, check in the story

Weightage in Board Exams

Quadratic Equations belongs to the **Algebra** unit of Class 10 Maths, with Polynomials, Pair of Linear Equations in Two Variables, and Arithmetic Progressions. Check the current CBSE course structure for the marks given to the unit; within it, this chapter is assessed in a predictable way.

Question type	What is usually asked
MCQ and Assertion-Reason	Nature of roots; k for equal roots; finding k from a given root; equations that look quadratic but are not
Short answers	Factorisation, including surd coefficients; the quadratic formula with a surd answer; age or number problems
Long answer	Speed and time, boat and stream, or taps and work: form, solve and reject a root
Case-based	A real situation split into parts: form the equation, solve, and use the discriminant to test a value

In a word problem, the key steps are forming the equation correctly and giving a reason when you reject a root. Practise every NCERT exercise question first, then the speed, boat and taps types above.



Got this PDF from a friend?

Everything here is free at **chapternotes.in** — 134 chapters of notes, a gamified practice app, and full NEET mock & prediction papers.



Scan for the full chapter page with practice questions

Join our Telegram · daily NEET PYQ with solution + every new PDF drop · t.me/chapternotes_in