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Number System Class 9 Notes | CBSE Maths Chapter 3

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Number System Class 9 Notes | CBSE Maths Chapter 3

Number System is Chapter 3 of CBSE Class 9 Maths. In the new 2026-27 NCERT book it appears as "The World of Numbers", and it traces how numbers grew from counting pebbles to the complete real number line: natural numbers, zero, integers, rational numbers, irrational numbers such as $\sqrt{2}$ and π , and finally the real numbers with their decimal expansions.

The chapter carries 7 marks in the Class 9 paper. School exams test it through rational number arithmetic, placing numbers on the number line, finding rational numbers between two given numbers, the proof that $\sqrt{2}$ or $\sqrt{3}$ is irrational, and converting repeating decimals into p/q form. The irrationality proofs return in Class 10 Real Numbers, so a firm grip here pays off in the board year.

Quick Answer - Number System in 30 Seconds

Natural numbers $\{1, 2, 3, \dots\}$ came from counting. Brahmagupta made zero a number (628 CE) and added negative numbers, giving the integers $\mathbb{Z}$. A **rational number** is any p/q with p, q integers and $q \neq 0$; rationals are closed under $+$, $-$, $\times$ and $\div$ (except by 0) and are dense, so the average $(a + b)/2$ always lies between a and b . **Irrational numbers** such as $\sqrt{2}$ and π cannot be written as p/q ; their decimals never end and never repeat. Rational decimals terminate or repeat. Rationals and irrationals together form the **real numbers** $\mathbb{R}$.

All Key Formulas at a Glance

Idea	Rule or Formula	Remember
Rational number	p/q , p and q integers, $q \neq 0$	Lowest form: p , q co-prime
Equality of rationals	$a/b = c/d$ if $ad = bc$	Cross-multiply to check
Addition and subtraction	$a/b \pm c/b = (a \pm c)/b$	Make denominators equal first
Multiplication	$(a/b) \times (c/d) = ac/bd$	$b \neq 0$, $d \neq 0$
Division	$(a/b) \div (c/d) = (a/b) \times (d/c) = ad/bc$	b, c, d all $\neq 0$
Distributive law	$p(q + r) = pq + pr$	Holds for all rationals
Absolute value and distance	$ x \geq 0$; distance between a and $b = a - b $	$ -5/3 = 5/3$
Rational between a and b	$(a + b)/2$	Proves density
Terminating decimal test	p/q in lowest form terminates exactly when $q = 2^m \times 5^n$	Any other prime factor: repeats
Pure repeating to p/q	Multiply by 10^n (n = repeating digits), subtract	$0.4545... = 45/99 = 5/11$
General repeating to p/q	Multiply by 10^m , then by 10^n more, subtract	$0.1666... = 15/90 = 1/6$
Diagonal of unit square	$1^2 + 1^2 = d^2$, so $d = \sqrt{2}$	Construct $\sqrt{n}$ with a spiral

Key Concepts

1. Natural Numbers and the Need to Count

Numbers began with a practical need: keeping count. Picture a herder in a settlement on the banks of the Saraswati river thousands of years ago. Every morning the cattle go out to graze and every evening they come back. With no number words and no written symbols, the herder still needs to know if a calf has wandered off.

The answer is **one-to-one correspondence**. For every cow that leaves, the herder drops one pebble into a clay pot. For every cow that returns, one pebble comes out. An empty pot in the evening means the herd is safe. Pebbles left over mean cows are missing. Matching one object to exactly one other object in this way is the idea behind the **natural numbers**:

$$\mathbf{N} = \{1, 2, 3, 4, \dots\}$$

Evidence written in bone. The earliest physical records of counting are tally marks carved into bone.

- The **Lebombo Bone**, found in the Lebombo Mountains between South Africa and Swaziland, is about 35,000 years old. It has 29 deliberately carved notches of uniform size. It may have been a lunar phase counter.
- The **Ishango bone**, found near the headwaters of the Nile in the Democratic Republic of Congo, dates to around 20,000 BCE. It has three columns of notches. One column groups them as 11, 13, 17 and 19, which are exactly the prime numbers between 10 and 20. Another column seems to show doubling (multiplication by 2).

The Indian context. The Vedas gave names to every power of 10 up to 10^{12} , which was called **parārdha**. In the Lalitavistara (4th century BCE), the Buddha describes names up to 10^{53} , called **tallakṣhaṇa**. Expressing quantities as powers of 10 in the Ṛigveda set the stage for the place-value decimal system developed in India, the system the whole world uses today. That system also paved the way for zero.

Closure. A set is **closed** under an operation if doing that operation on any two members always gives a member of the same set.

Operation on natural numbers	Closed?	Reason
Addition	Yes	The sum of two natural numbers is a natural number: $5 + 4 = 9$
Multiplication	Yes	$6 \times 7 = 42$, again a natural number
Subtraction	No	$4 - 9 = -5$ and $5 - 5 = 0$, neither is a natural number
Division	No	$3 \div 4 = 3/4$, which is a fraction

One counter-example is enough to show that a set is not closed. This gap in subtraction led to zero and the negative numbers.

2. Zero (Śhūnya): When Nothing Became a Number

For a very long time the number line started at 1. If you had five apples and gave all five away, there was no number for what you had left. The Babylonians and the Mayans used placeholder symbols to mark an empty column in a number, but they did not treat “nothing” as a number you could add, subtract or multiply.

From philosophy to mathematics. In the Upanishads and in Buddhist literature, **śhūnyatā** (emptiness) described the still mind sought in meditation. Because Indian thinkers valued emptiness, the idea moved into mathematics through Āryabhaṭa and Brahmagupta.

The symbol. The **Bakhshālī Manuscript**, dated to the early centuries CE, uses a bold dot called a **bindu** to stand for zero.

The rules. A symbol becomes a number only when it has rules. In the **Brāhmasphuṭasiddhānta (628 CE)**, Brahmagupta defined zero as the result of subtracting a number from itself: **$a - a = 0$** . He then gave the laws of arithmetic with zero:

Brahmagupta's rule	In symbols	Example
Adding zero leaves a number unchanged	$a + 0 = a$	$17 + 0 = 17$
Subtracting zero leaves a number unchanged	$a - 0 = a$	$9 - 0 = 9$
Any number multiplied by zero gives zero	$a \times 0 = 0$	$250 \times 0 = 0$

Adding zero to the natural numbers gives the **whole numbers** $\{0, 1, 2, 3, \dots\}$. Division by zero is never allowed: there is no number that, multiplied by 0, gives a non-zero answer. You will see this rule again in the definition of a rational number.

3. Integers: Fortunes, Debts and the Sign Rules

Since $5 - 5 = 0$, Brahmagupta asked what $3 - 5$ should be, and found the answer in trade. He described two states:

- **Fortunes (Dhana):** positive numbers, standing for wealth or assets.
- **Debts (Riṇa):** negative numbers, standing for money owed.

Moving to the left of zero on the number line, he formally introduced negative numbers. Positive natural numbers, their negatives and zero together form the **integers**:

$$\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$$

The letter Z comes from the German word *Zahlen*, meaning numbers. On the number line, debts lie to the left of zero and fortunes to the right, with each integer the same distance from the next.

Brahmagupta's rules for integers (we still use them exactly as he stated them over 1,300 years ago):

Rule in words	Example	Real-life meaning
A fortune plus a fortune is a fortune	$5 + 4 = 9$	Earnings add up
A debt plus a debt is a debt	$(-5) + (-4) = -9$	Owe ₹5, borrow ₹4 more, you owe ₹9
Fortune minus zero is a fortune; debt minus zero is a debt	$7 - 0 = 7$; $-6 - 0 = -6$	Nothing taken away, nothing changes
Product of a debt and a fortune is a debt	$(-3) \times 4 = -12$	Four debts of ₹3 make a debt of ₹12
Product of two debts is a fortune	$(-3) \times (-4) = 12$	Removing four debts of ₹3 leaves you ₹12 richer

Why negative $\times$ negative is positive. A negative number is a debt. Multiplying by a negative number means removing that debt. If someone takes away four of your debts of ₹3 each, you are ₹12 better off. So $(-3) \times (-4) = +12$.

Subtracting a negative. $10 - (-5) = 15$. If you have ₹10 and someone cancels a debt of ₹5 you owed, your position improves by ₹5. Subtracting a debt is the same as adding a fortune: $a - (-b) = a + b$.

Division follows the same sign pattern. $(-20) \div 4 = -5$ and $(-20) \div (-4) = 5$. Like signs give a positive answer; unlike signs give a negative answer.

Integers are closed under addition, subtraction and multiplication. They are not closed under division, because $3 \div 4$ is not an integer. That gap leads to fractions.

4. Rational Numbers and Their Arithmetic

Measuring needs parts of a whole: half a cup of ghee, a field shared among three children. Numbers that stand for parts of a whole are **fractions**. Every positive fraction has an additive inverse, a **negative fraction**: $-3/4$ for $3/4$, $-19/7$ for $19/7$. The minus sign may sit in front, on the numerator or on the denominator: $-1/5 = (-1)/5 = 1/(-5)$.

All integers and all fractions, positive and negative, together form the **rational numbers**, written **Q** (for quotient).

Definition: A rational number is any number that can be expressed in the form **p/q**, where p and q are integers and **q ≠ 0**.

The reason for q ≠ 0. p/q means the number which multiplied by q gives p. If q = 0, that number would have to satisfy (number) × 0 = p. By Brahmagupta's rule, anything times 0 is 0, so no number works when p ≠ 0, and every number works when p = 0. Either way p/0 has no single value, so it is excluded.

Three important observations from NCERT:

- Every integer is rational.** $5 = 5/1$ and $-10 = -10/1$. So Q contains the natural numbers, the whole numbers and the integers.
- A rational number has many forms.** $-1/3 = -2/6 = -3/9 = -10/30 = -2026/6078$. These are **equivalent rational numbers**. You may divide the numerator and denominator by any common factor: $12/30 = 2/5$ (dividing both by 6).
- Standard (lowest) form.** When we speak of p/q or mark it on the number line, we assume q ≠ 0 and that p and q have no common factor other than 1, that is, p and q are **co-prime**. Among $1/2, 2/4, 3/6, 6/12, \dots, 1013/2026$, we choose $1/2$ to represent all of them.

Brahmagupta's laws for rational numbers:

Law	Rule	Condition
Equality	$a/b = c/d$ if $ad = bc$	$b, d \neq 0$
Addition	$a/b + c/b = (a + c)/b$	Write both with the same denominator first
Subtraction	$a/b - c/b = (a - c)/b$	Same denominator first
Multiplication	$(a/b) \times (c/d) = ac/bd$	$b \neq 0, d \neq 0$
Division	$(a/b) \div (c/d) = (a/b) \times (d/c) = ad/bc$	$b \neq 0, d \neq 0, c \neq 0$
Commutative	$a/b + c/d = c/d + a/b$ and $(a/b) \times (c/d) = (c/d) \times (a/b)$	All rationals
Distributive	$p(q + r) = pq + pr$	p, q, r rational

Making denominators equal. To add $7/12$ and $5/8$, find the LCM of 12 and 8, which is 24. Then $7/12 = 14/24$ and $5/8 = 15/24$, so the sum is $29/24$.

Closure of Q. Adding, subtracting or multiplying two rational numbers always gives a rational number. Dividing one rational number by another also gives a rational number, as long as you do not divide by zero. So Q is closed under all four operations, with the single exception of division by zero.

Set	+	-	×	÷ (non-zero divisor)
Natural numbers N	Closed	Not closed	Closed	Not closed
Integers Z	Closed	Closed	Closed	Not closed
Rational numbers Q	Closed	Closed	Closed	Closed

5. Rational Numbers on the Number Line, Absolute Value and Distance

Integers on the line. Choose a point and call it 0, the **origin**. One unit to the right is 1, two units is 2, and so on. One unit to the left is -1, two units is -2. Consecutive integers are equally spaced.

Rationals lie between integers. $1/2$ is exactly halfway between 0 and 1. $-3/4$ lies between -1 and 0.

Method to mark p/q ($q > 0$):

1. Divide the unit interval (the gap between two consecutive integers) into **q equal parts**.
2. Move **p parts** from 0: to the right if the number is positive, to the left if it is negative.

Example: $3/4$. Split the gap from 0 to 1 into four equal parts; the marks are $1/4$, $2/4 = 1/2$ and $3/4$. Move three parts to the right of 0.

Fractions greater than 1. $9/4 = 2 \frac{1}{4}$, so it lies between 2 and 3. Split the gap from 2 to 3 into four equal parts and move one part to the right of 2.

Absolute value. The absolute value of a rational number x, written **|x|**, is its distance from 0 on the number line.

- $|5/3| = 5/3$, $|-5/3| = 5/3$ and $|0| = 0$.
- The absolute value of a positive number is the number itself; for a negative number it is its positive value.
- So **$|x| \geq 0$** for every rational x.

Distance between two numbers. For rationals a and b, the distance between them on the number line is **$|a - b|$** . The distance between 3 and -4 is $|3 - (-4)| = |7| = 7$. The order does not matter: $|-4 - 3| = |-7| = 7$.

6. Density of Rational Numbers

Between the integers 1 and 2 there is no other integer, but there is the rational $3/2$. Between 1 and $3/2$ there is $5/4$. This never stops. Rational numbers are **dense**: between any two rational numbers, however close, there is always another rational number.

The averaging method. For rationals a and b with $a < b$, the average $(a + b)/2$ is rational and lies strictly between them.

- **Why it is rational:** $a + b$ is rational (closure under addition), and dividing a rational by the non-zero rational 2 gives a rational (closure under division).
- **Why it lies between:** since $a < b$, adding a to both sides gives $2a < a + b$, so $a < (a + b)/2$. Adding b to both sides of $a < b$ gives $a + b < 2b$, so $(a + b)/2 < b$.

Example: between 1 and $3/2$, the average is $(1 + 3/2)/2 = (5/2)/2 = 5/4$. Repeat the process with 1 and $5/4$ to get $9/8$, and so on without end. So there are **infinitely many rational numbers between any two rational numbers**.

The common denominator method (fast when you need many numbers). To find 5 rationals between $2/5$ and $3/5$, rewrite both with a larger denominator so that the numerators are far apart. Multiply numerator and denominator by 6: $2/5 = 12/30$ and $3/5 = 18/30$. Now $13/30$, $14/30$, $15/30$, $16/30$ and $17/30$ all lie between them. To find n numbers this way, the two numerators must differ by at least $n + 1$.

7. Irrational Numbers: $\sqrt{2}$, $\sqrt{3}$ and π

Take a square whose sides are each 1 unit long. By the **Baudhāyana-Pythagoras Theorem** the diagonal d satisfies $1^2 + 1^2 = d^2$, so $d^2 = 2$ and $d = \sqrt{2}$. Baudhāyana met such lengths while writing his Śhulbasūtra, a manual for building fire altars, around 800 BCE. The ancient Greeks met the same problem a few centuries later.

Definition: Numbers on the number line that cannot be expressed as a ratio of integers p/q ($q \neq 0$) are called **irrational numbers**. The set is often written I .

Proof that $\sqrt{2}$ is irrational (proof by contradiction). The first proof is credited to **Hippasus**, a member of the Pythagorean school (c. 400 BCE, as NCERT dates it). In a proof by contradiction you assume the opposite of what you want to prove and show that the assumption leads to something impossible.

1. **Assume $\sqrt{2}$ is rational.** Then $\sqrt{2} = p/q$, where p and q are integers, $q \neq 0$, and p and q are co-prime (the fraction is in lowest form).
2. **Square both sides:** $2 = p^2/q^2$.
3. **Multiply by q^2 :** $2q^2 = p^2$.
4. **p is even.** p^2 equals 2 times an integer, so p^2 is even. If the square of an integer is even, the integer itself is even. So p is even; write $p = 2k$ for some integer k .
5. **Substitute:** $2q^2 = (2k)^2 = 4k^2$.
6. **Divide by 2:** $q^2 = 2k^2$.
7. **q is even.** q^2 is 2 times an integer, so q^2 is even and q is even.
8. **Contradiction.** p and q are both even, so they share the factor 2. This contradicts step 1, where p and q had no common factor other than 1.

Every step after the assumption is correct, so the assumption must be wrong. $\therefore \sqrt{2}$ is irrational.

Why an even square means an even number: if $p = 2m + 1$ were odd, $p^2 = 4m^2 + 4m + 1$ would be odd. The same argument with the prime 3 in place of 2 proves $\sqrt{3}$ irrational, and it extends to $\sqrt{5}$, $\sqrt{7}$ and $\sqrt{10}$. It fails for $\sqrt{4}$ and $\sqrt{9}$, which equal 2 and 3.

Constructing $\sqrt{2}$ on the number line.

1. On the number line mark O at 0 and A at 1, so $OA = 1$ unit. Draw a line perpendicular to OA through A.
2. On this perpendicular mark B with $AB = 1$ unit. Join OB. Triangle OAB is right-angled at A, so $OB = \sqrt{(1^2 + 1^2)} = \sqrt{2}$ units.
3. With O as centre and radius OB, draw an arc that cuts the number line at P. Then $OP = OB = \sqrt{2}$, so P represents $\sqrt{2}$. Drawing the arc on the other side of O gives $-\sqrt{2}$.

The square root spiral. Repeat the idea. At B draw a perpendicular to OB and mark C with $BC = 1$ unit. Then $OC = \sqrt{((\sqrt{2})^2 + 1^2)} = \sqrt{3}$. At C draw a perpendicular to OC, mark D with $CD = 1$, and $OD = \sqrt{4} = 2$. Continuing gives hypotenuses $\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}, \dots$, and in general the n -th triangle has hypotenuse $\sqrt{(n + 1)}$. The shape formed is the **square root spiral**. Any $\sqrt{n}$ can be transferred to the number line with a compass centred at O.

The story of π . π is the ratio of a circle's circumference to its diameter. Āryabhaṭa (499 CE) gave the approximation **3927/1250 = 3.1416** and called it **āsanna** (approximate), hinting that no exact fraction could be found. π was proved irrational by **Lambert in 1761**. In the 14th century **Mādhava of Sangamagrama**, founder of the Kerala School of Mathematics, found an exact formula as an infinite sum:

$$\pi = 4 \times (1 - 1/3 + 1/5 - 1/7 + \dots)$$

Adding infinitely many terms means finding the value that the running total gets closer and closer to.

8. Real Numbers and Decimal Expansions

The dense rationals together with the irrationals fill the number line completely. This unbroken line is the set of **real numbers R**. Every real number is a point on the line, and every point on the line is a real number.

How the sets fit together: N is contained in Z, Z is contained in Q, and Q together with the irrationals I makes up R. The irrationals are separate from Q: no number is both rational and irrational.

Decimal expansion of a rational number. Divide the numerator by the denominator. Exactly one of two things happens:

1. **It terminates:** the remainder becomes 0 and the decimal stops. $3/8 = 0.375$.
2. **It repeats:** the remainder never becomes 0, and a block of digits loops forever. $5/11 = 0.454545\dots$, written with a bar over the repeating block 45.

Why a rational decimal must repeat if it does not stop. When dividing by 7, the possible non-zero remainders are only 1, 2, 3, 4, 5 and 6. Among any seven successive remainders, two must be the same, and from then on the division repeats itself. So the repeating block of $1/7$ is at most 6 digits long. In fact $1/7 = 0.142857142857\dots$, with block 142857.

Predicting the type without dividing. Write p/q in lowest form and factorise q .

- If the prime factors of q are **only 2, only 5, or only 2 and 5**, the decimal **terminates**. The reason: you can multiply numerator and denominator by a suitable number to make the denominator a power of 10. For example, $3/20 = 3/(2^2 \times 5) = (3 \times 5)/(2^2 \times 5^2) = 15/100 = 0.15$.
- If q has **any other prime factor** (3, 7, 11, 13, ...), the decimal is **non-terminating and repeating**. $4/15$ has $15 = 3 \times 5$, so it repeats: $4/15 = 0.2666\dots$

- **Number of decimal places.** If $q = 2^m \times 5^n$, the decimal ends after the larger of m and n places. $7/40$ has $40 = 2^3 \times 5$, so it has 3 decimal places: $7/40 = 175/1000 = 0.175$.

Converting decimals to p/q.

Type	Steps	Example
Terminating	Write over the matching power of 10 and simplify	$0.35 = 35/100 = 7/20$
Pure repeating (block starts right after the point)	Let $x =$ the decimal. Multiply by 10^n , $n =$ number of repeating digits. Subtract x . Solve.	$x = 0.666\dots$; $10x = 6.666\dots$; $9x = 6$; $x = 2/3$
General repeating (some digits before the block)	Let $x =$ the decimal. Multiply by 10^m , $m =$ number of non-repeating digits. Multiply that by 10^n , $n =$ number of repeating digits. Subtract the two shifted equations. Solve.	$x = 0.1666\dots$; $10x = 1.666\dots$; $100x = 16.666\dots$; $90x = 15$; $x = 1/6$

Cyclic numbers. The block of $1/7$ is 142857. Multiply it by 1 to 6:

$$142857 \times 1 = 142857, \times 2 = 285714, \times 3 = 428571, \times 4 = 571428, \times 5 = 714285, \times 6 = 857142.$$

The same six digits appear each time, only rotated in a circle. A number with this property is a **cyclic number**. It reflects the fact that $2/7$, $3/7$, ..., $6/7$ have repeating blocks that are rotations of the block of $1/7$.

Decimal expansion of an irrational number. It never ends and never repeats. $\sqrt{2} = 1.4142135623730950488\dots$ and $\pi = 3.1415926535897932384\dots$. A decimal such as $1.01001000100001\dots$, where the number of zeros keeps growing, has a clear pattern but no fixed block that repeats, so it is irrational.

$0.999\dots = 1$. Let $x = 0.999\dots$. Then $10x = 9.999\dots$. Subtracting, $9x = 9$, so $x = 1$. Any terminating decimal also has a form ending in repeating 9s: $2.47 = 2.46999\dots$. So a decimal representation need not be unique.

Formula and Theorem Sheet

Result	Statement or formula	When to use
Brahmagupta's zero rules	$a + 0 = a$; $a - 0 = a$; $a \times 0 = 0$; $a - a = 0$	Arithmetic with zero
Sign rules	$(+)(+) = +$; $(-)(-) = +$; $(+)(-) = -$; $a - (-b) = a + b$	Integer and rational arithmetic
Rational number	p/q , $p, q \in \mathbb{Z}$, $q \neq 0$	Classifying numbers
Equality test	$a/b = c/d$ exactly when $ad = bc$	Proving two rationals equal
Multiplication	$(a/b)(c/d) = ac/bd$	$b, d \neq 0$
Division	$(a/b) \div (c/d) = ad/bc$	$b, c, d \neq 0$
Density	$a < (a + b)/2 < b$ for $a < b$	Rational between two rationals
Irrationality of $\sqrt{2}$, $\sqrt{3}$	Proof by contradiction using co-prime p, q	3-mark and 5-mark proofs
Baudhāyana-Pythagoras	Hypotenuse ² = base ² + perpendicular ²	Constructing $\sqrt{n}$, square root spiral
Terminating test	p/q (lowest form) terminates exactly when $q = 2^m 5^n$	Predicting decimal type
Pure repeating	For $x = 0.(\text{block of } n \text{ digits})$: $(10^n - 1)x = \text{the block}$	$0.abab\ldots = ab/99$
General repeating	$(10^{m+n} - 10^m)x = \text{difference of shifted numbers}$	$0.1666\ldots = 15/90$

Important Definitions

Term	Definition
One-to-one correspondence	Matching each object of one collection with exactly one object of another, as in one pebble for each cow
Natural numbers (N)	The counting numbers {1, 2, 3, 4, ...}
Zero (śhūnya)	The number obtained by subtracting a number from itself, $a - a = 0$ (Brahmagupta, 628 CE)
Integers (Z)	Positive natural numbers, their negatives and zero: {..., -2, -1, 0, 1, 2, ...}; Z from German <i>Zahlen</i>
Rational number (Q)	A number expressible as p/q where p and q are integers and $q \neq 0$
Absolute value $ x $	The distance of x from 0 on the number line; always ≥ 0
Density of rationals	Between any two rational numbers there is another rational number, hence infinitely many
Irrational number	A number on the number line that cannot be written as p/q with p, q integers and $q \neq 0$; its decimal never ends and never repeats
Real numbers (R)	All rational and irrational numbers together; they fill the number line with no gaps

Solved Examples (NCERT-Based)

Example 1: Proving two rationals equal (Exercise Set 3.3 type)

Show that (i) $2/3 = 4/6$ and (ii) $-3/5 = -6/10$.

Solution: Use $a/b = c/d$ exactly when $ad = bc$.

(i) $2 \times 6 = 12$ and $3 \times 4 = 12$. The cross products match, so $2/3 = 4/6$.

(ii) $(-3) \times 10 = -30$ and $5 \times (-6) = -30$. Equal, so $-3/5 = -6/10$.

Example 2: Adding rationals with different denominators

Find (i) $7/12 + 5/8$ and (ii) $-4/7 + 3/14$.

Solution: (i) LCM of 12 and 8 is 24. $7/12 = 14/24$ and $5/8 = 15/24$. Sum = **29/24** (lowest form, since 29 is prime and does not divide 24).

(ii) LCM of 7 and 14 is 14. $-4/7 = -8/14$. Sum = $(-8 + 3)/14 =$ **-5/14**.

Example 3: Product and quotient

Find (i) $7/11 \times 5/8$ and (ii) $-4/7 \div 5/14$.

Solution: (i) $(7 \times 5)/(11 \times 8) = \mathbf{35/88}$.

(ii) Multiply by the reciprocal: $-4/7 \times 14/5 = (-4 \times 14)/(7 \times 5) = -56/35$. Divide both by 7: **$-8/5$** . Cancelling 14 with 7 before multiplying ($14/7 = 2$) gives $-4 \times 2/5 = -8/5$ directly.

Example 4: Verifying the distributive law

Show that $(1/2 + 3/4) \times 8/3 = 1/2 \times 8/3 + 3/4 \times 8/3$.

Solution: LHS: $1/2 + 3/4 = 2/4 + 3/4 = 5/4$. Then $5/4 \times 8/3 = 40/12 = 10/3$.

RHS: $1/2 \times 8/3 = 8/6 = 4/3$. $3/4 \times 8/3 = 24/12 = 2$. Sum = $4/3 + 2 = 4/3 + 6/3 = 10/3$.

LHS = RHS = $10/3$. The distributive law holds.

Example 5: Finding an unknown rational (End-of-Chapter type)

If $x/3 + x/5 = 16/15$, find the rational number x .

Solution: LCM of 3 and 5 is 15. $x/3 + x/5 = 5x/15 + 3x/15 = 8x/15$.

So $8x/15 = 16/15$. Multiply both sides by 15: $8x = 16$, so **$x = 2$** .

Example 6: The tailor and the silk (Exercise Set 3.4)

A tailor has $15 \frac{3}{4}$ metres of silk. One kurta needs $2 \frac{1}{4}$ metres. How many kurtas can he make?

Solution: Convert to improper fractions. $15 \frac{3}{4} = (60 + 3)/4 = 63/4$. $2 \frac{1}{4} = 9/4$.

Number of kurtas = $63/4 \div 9/4 = 63/4 \times 4/9 = 63/9 = \mathbf{7 \text{ kurtas}}$, with no silk left over.

Example 7: Rational numbers between two rationals

(i) Find three rational numbers between $-1/2$ and $1/4$. (ii) Find five rational numbers between $2/5$ and $3/5$.

Solution: (i) Averaging: $(-1/2 + 1/4)/2 = (-1/4)/2 = -1/8$. Average of $-1/2$ and $-1/8$: $(-4/8 - 1/8)/2 = -5/16$. Average of $-1/8$ and $1/4$: $(-1/8 + 2/8)/2 = 1/16$. So **$-5/16$, $-1/8$ and $1/16$** lie between $-1/2$ and $1/4$.

(ii) Multiply numerator and denominator by 6: $2/5 = 12/30$ and $3/5 = 18/30$. The numbers **$13/30$, $14/30$, $15/30$, $16/30$, $17/30$** lie between them. In lowest form these are $13/30$, $7/15$, $1/2$, $8/15$ and $17/30$.

Example 8: Absolute value and distance

Find the distance between (i) -4 and 3 , (ii) $-3/4$ and $5/6$ on the number line.

Solution: (i) $|3 - (-4)| = |7| = 7$ units.

(ii) $|-3/4 - 5/6| = |-9/12 - 10/12| = |-19/12| = 19/12$ units. Distances are never negative, so the absolute value removes the minus sign.

Example 9: Predicting terminating or repeating (Exercise Set 3.5)

Without long division, decide which of $7/20$, $4/15$ and $13/250$ terminate. Then check.

Solution: All three are in lowest form.

- $20 = 2^2 \times 5$. Only 2 and 5, so $7/20$ **terminates**. Check: $7/20 = 35/100 = 0.35$.
- $15 = 3 \times 5$. The factor 3 means $4/15$ is **non-terminating repeating**. Check: $4/15 = 0.2666\ldots$
- $250 = 2 \times 5^3$. Only 2 and 5, so $13/250$ **terminates**. Check: multiply by 4/4: $52/1000 = 0.052$.

Example 10: Converting a general repeating decimal

Express $0.23535\ldots$ (the block 35 repeats, the 2 does not) in the form p/q .

Solution: Let $x = 0.23535\ldots$. One non-repeating digit, so multiply by 10: $10x = 2.3535\ldots$ (equation 1).

Two repeating digits, so multiply equation 1 by 100: $1000x = 235.3535\ldots$ (equation 2).

Subtract equation 1 from equation 2: $1000x - 10x = 235.3535\ldots - 2.3535\ldots$, so $990x = 233$.

$x = 233/990$. 233 is prime and does not divide 990, so this is the lowest form.

Example 11: Classifying numbers (Exercise Set 3.5)

Classify as rational or irrational, and give p/q for the rational ones: (i) $\sqrt{81}$ (ii) $\sqrt{12}$ (iii) $0.33333\ldots$ (iv) $0.123451234512345\ldots$ (v) $1.01001000100001\ldots$ (vi) 23.560185612239874790120

Solution:

1. $\sqrt{81} = 9 = 9/1$. **Rational**.
2. $\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}$. Since $\sqrt{3}$ is irrational, $2\sqrt{3}$ is irrational (if $2\sqrt{3} = p/q$ then $\sqrt{3} = p/2q$ would be rational). **Irrational**.
3. $x = 0.333\ldots$; $10x = 3.333\ldots$; $9x = 3$; $x = 1/3$. **Rational**.
4. Block 12345 repeats (5 digits). $100000x - x = 12345$, so $x = 12345/99999 = 4115/33333$ (dividing by 3). **Rational**.
5. The number of zeros keeps growing, so no fixed block repeats. **Irrational**.
6. The decimal stops, so it is terminating: $23560185612239874790120/10^{21}$. **Rational**.

Example 12: Proving $\sqrt{3}$ is irrational

Prove that $\sqrt{3}$ is irrational.

Solution: Suppose $\sqrt{3}$ is rational. Then $\sqrt{3} = p/q$, where p, q are integers, $q \neq 0$, and p, q are co-prime.

Squaring: $3 = p^2/q^2$, so $3q^2 = p^2$. Hence 3 divides p^2 . Since 3 is prime, 3 divides p . Write $p = 3k$.

Then $3q^2 = 9k^2$, so $q^2 = 3k^2$. Hence 3 divides q^2 , and so 3 divides q .

Now 3 is a common factor of p and q , which contradicts the fact that p and q are co-prime. The assumption was false. $\therefore \sqrt{3}$ is irrational.

Competency-Based Questions (with answers)

1. Case-based: The Lothal spice trade

A merchant in the port city of Lothal exchanges bags of spices for copper ingots. He receives 15 ingots for every 2 bags. A second merchant pays in debts and fortunes: he takes a loan of ₹850, makes a profit of ₹1,200 the next day, and a loss of ₹450 the following week.

(a) How many ingots does the first merchant get for 12 bags? 12 bags = 6 groups of 2 bags, so $6 \times 15 = \mathbf{90}$ ingots.

(b) Write the second merchant's record as one integer expression and find his final position. $(-850) + 1200 + (-450) = 350 - 450 = \mathbf{-100}$. He is in debt by ₹100.

2. Case-based: Night in Ladakh

The temperature in a high-altitude desert in Ladakh is 4°C at noon. By midnight it has dropped by 15°C .

(a) Find the midnight temperature. $4 - 15 = \mathbf{-11^\circ\text{C}}$.

(b) How far apart are the noon and midnight readings on the number line? $|4 - (-11)| = 15$ degrees, which matches the drop.

(c) Is the set of natural numbers enough to record these temperatures? Justify. No. -11 is a negative integer. The natural numbers are not closed under subtraction ($4 - 15$ is outside $\mathbb{N}$), so integers are needed.

3. Source-based: The Ishango bone

One column of the Ishango bone (about 20,000 BCE) groups its notches as 11, 13, 17 and 19. Another column appears to show doubling.

(a) What do 11, 13, 17 and 19 have in common? They are all the prime numbers between 10 and 20; each has exactly two factors, 1 and itself.

(b) Give the next three numbers that fit this pattern. **23, 29 and 31**, the next three primes.

4. Assertion-Reason

Assertion (A): The decimal expansion of $13/250$ terminates. **Reason (R):** The denominator of a rational number in lowest form that has only 2 and 5 as prime factors can be turned into a power of 10.

Answer: (a) Both A and R are true, and R is the correct explanation of A. $250 = 2 \times 5^3$; multiplying by $2^2/2^2$ gives $52/1000 = 0.052$.

5. Assertion-Reason

Assertion (A): Between any two rational numbers there are infinitely many rational numbers. **Reason (R):** The average of two rational numbers is a rational number lying strictly between them.

Answer: (a) Both A and R are true, and R is the correct explanation of A. Taking averages again and again produces a new rational each time, without end.

6. Assertion-Reason

Assertion (A): $22/7$ is an irrational number. **Reason (R):** π is irrational, and $22/7$ is often used for π .

Answer: (d) A is false, R is true. $22/7$ is a ratio of two integers, so it is rational. It is only an approximation of π , which Lambert proved irrational in 1761.

7. Data reasoning: Cyclic patterns in $1/13$

A student performs long division and gets $1/13 = 0.076923076923\dots$, $2/13 = 0.153846153846\dots$, $3/13 = 0.230769230769\dots$ and $4/13 = 0.307692307692\dots$

(a) What is the repeating block of $1/13$ and its length? 076923, six digits long.

(b) Which of $2/13$, $3/13$, $4/13$ have blocks that are rotations of 076923? $3/13$ (230769) and $4/13$ (307692) are rotations. $2/13$ has block 153846, which uses different digits, so it belongs to a second cycle.

(c) Why can the block of any fraction with denominator 13 be at most 12 digits long? The only possible non-zero remainders when dividing by 13 are 1 to 12. Among any 13 successive remainders, two must be equal, and the division then loops. So the block has at most 12 digits.

8. Source-based: Mādhava's series

Mādhava of Sangamagrama wrote $\pi = 4 \times (1 - 1/3 + 1/5 - 1/7 + \dots)$. Āryabhaṭa earlier gave $\pi \approx 3927/1250$.

(a) Find $4 \times (1 - 1/3 + 1/5)$. $1 - 1/3 + 1/5 = 15/15 - 5/15 + 3/15 = 13/15$. Times 4 gives $52/15 \approx 3.467$.

(b) Write $3927/1250$ as a decimal and say whether it is exact. $1250 = 2 \times 5^4$, so it terminates: $3927/1250 = 31416/10000 = 3.1416$. It is an approximation (āsanna); π itself is irrational.

(c) Why did Mādhava need infinitely many terms? Every finite sum of fractions is a rational number, and π is irrational. No finite sum can equal π , so an infinite sum is required.

Important Questions for Exams

1-Mark Questions

1. **Is zero a rational number?** Yes. $0 = 0/1$, with 0 and 1 integers and $1 \neq 0$.
2. **Find $|-7/9|$.** $7/9$.
3. **Write the additive inverse of $-19/7$.** $19/7$, because $-19/7 + 19/7 = 0$.
4. **Which of these is irrational: $\sqrt{16}$, $0.1212\dots$, $\sqrt{7}$, $22/7$?** $\sqrt{7}$. ($\sqrt{16} = 4$; $0.1212\dots = 12/99 = 4/33$; $22/7$ is a ratio of integers.)
5. **Who defined zero as a - a and in which work?** Brahmagupta, in the Brāhmasphuṭasiddhānta (628 CE).

2-Mark Questions

1. **Without division, state whether $18/125$ has a terminating decimal. If so, find the number of decimal places.**
 $18/125$ is in lowest form and $125 = 5^3$, so it terminates. Multiply by $2^3/2^3$: $144/1000 = 0.144$, which has **3 decimal places**.
2. **Express $0.4545\dots$ in the form p/q .** $x = 0.4545\dots$; $100x = 45.4545\dots$; $99x = 45$; $x = 45/99 = \mathbf{5/11}$.
3. **Find 6 rational numbers between 3 and 4.** Write $3 = 21/7$ and $4 = 28/7$. Then $22/7, 23/7, 24/7, 25/7, 26/7, 27/7$ all lie between them.
4. **Let a and b be non-zero rationals with $a + 1/b = 0$. Without using values, decide whether ab is positive or negative.** $a = -1/b$, so $ab = (-1/b) \times b = -1$. ab is **negative**.

3-Mark Questions

1. **Express $2.35777\dots$ (only 7 repeating) in the form p/q .** Let $x = 2.35777\dots$. Two non-repeating decimal digits: $100x = 235.777\dots$. One repeating digit: $1000x = 2357.777\dots$. Subtract: $900x = 2122$, so $x = 2122/900 = \mathbf{1061/450}$.
2. **Explain, with steps, how to locate $\sqrt{2}$ on the number line.** Mark $OA = 1$ unit on the number line with O at 0. Draw $AB \perp OA$ with $AB = 1$ unit. By Baudhāyana-Pythagoras, $OB = \sqrt{(1+1)} = \sqrt{2}$. With centre O and radius OB , draw an arc to cut the number line at P . $OP = \sqrt{2}$, so P represents $\sqrt{2}$.
3. **A rational number in lowest form has denominator $2^3 \times 5$. How many decimal places does its expansion have? Explain.** The denominator is 40. Multiply numerator and denominator by $5^2 = 25$: denominator becomes $1000 = 10^3$. The numerator p is co-prime to 40, so p is odd and not a multiple of 5; $25p$ ends in 5, so the last digit is non-zero. The decimal has exactly **3 decimal places**.

5-Mark Questions

1. **Prove that $\sqrt{5}$ is irrational. Hence explain why the same method fails for $\sqrt{9}$.** Assume $\sqrt{5} = p/q$ with p, q co-prime integers, $q \neq 0$. Squaring: $5q^2 = p^2$, so 5 divides p^2 and, since 5 is prime, 5 divides p . Write $p = 5k$: $5q^2 = 25k^2$, so $q^2 = 5k^2$, and 5 divides q . Then 5 is a common factor of p and q , contradicting co-primality. Hence $\sqrt{5}$ is irrational. For $\sqrt{9}$ the equation $9q^2 = p^2$ has the solution $p = 3, q = 1$ with p, q co-prime, so no contradiction appears: $\sqrt{9} = 3$ is rational.
2. **Three rational numbers x, y, z satisfy $x + y + z = 0$ and $xy + yz + zx = 0$. Show that $x = y = z = 0$.** Square the first equation: $(x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx) = 0$. Since $xy + yz + zx = 0$, we get $x^2 + y^2 + z^2 = 0$. Each square is ≥ 0 , and three non-negative numbers add to 0 only if each is 0. So $x^2 = y^2 = z^2 = 0$, which gives **$x = y = z = 0$** .

Common Mistakes and Examiner Tips

1. **Dropping the condition $q \neq 0$.** Writing "a rational number is p/q " without " p, q integers and $q \neq 0$ " loses the definition mark. Always write all three parts.
 2. **Calling $22/7$ or 3.14 equal to π .** Both are rational approximations. π is irrational. If a question asks you to classify $22/7$, the answer is rational.
 3. **Skipping "lowest form" in the irrationality proof.** The contradiction depends on p and q being co-prime. Say so in step 1, or the final step has nothing to contradict.
 4. **Copying the $\sqrt{2}$ wording into the $\sqrt{3}$ proof.** For $\sqrt{3}$ or $\sqrt{5}$, write " 3 divides p^2 , and 3 is prime, so 3 divides p ". The prime fact is what makes the step valid.
 5. **Testing the denominator before reducing.** $6/15$ looks like it repeats (15 has the factor 3), but $6/15 = 2/5 = 0.4$ terminates. Reduce to lowest form first, then factorise.
 6. **Using the wrong multiplier for repeating decimals.** Count repeating digits: one digit repeating uses 10 , two use 100 , three use 1000 . For a general repeating decimal, first shift past the non-repeating digits, then shift by one full block.
 7. **Adding fractions by adding tops and bottoms.** $1/2 + 1/3$ is $5/6$, never $2/5$. Make the denominators equal first.
 8. **Forgetting to flip the divisor.** $a/b \div c/d = a/b \times d/c$. Flip the second fraction only, and remember c must be non-zero.
 9. **Negative absolute values or distances.** $|x|$ and $|a - b|$ are never negative. If your distance comes out as -7 , you forgot the modulus.
 10. **Treating a patterned decimal as rational.** $1.01001000100001\dots$ has a pattern but no fixed repeating block, so it is irrational. Rational means a block that repeats unchanged.
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Quick Revision Points

- $N = \{1, 2, 3, \dots\}$; Brahmagupta (628 CE) made zero a number: $a - a = 0$, $a \times 0 = 0$
- Integers Z (from *Zahlen*): fortunes (dhana) positive, debts (ṛiṇa) negative
- Debt $\times$ fortune = debt; debt $\times$ debt = fortune; $a - (-b) = a + b$
- N is closed under $+$ and $\times$; Z under $+$, $-$, $\times$; Q under $+$, $-$, $\times$, $\div$ (divisor non-zero)
- Rational: p/q , p, q integers, $q \neq 0$; every integer $n = n/1$ is rational
- Lowest form: p, q co-prime; $a/b = c/d$ exactly when $ad = bc$
- Mark p/q : split the unit into q parts, move p parts right (positive) or left (negative)
- $|x|$ = distance from 0 , always ≥ 0 ; distance between a and $b = |a - b|$
- Density: $(a + b)/2$ lies between a and b ; infinitely many rationals between any two
- Irrational: not expressible as p/q ; decimal never ends, never repeats. Examples $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, π
- $\sqrt{2}$ irrational: Hippasus, proof by contradiction; p and q both come out even
- $\sqrt{2}$ on the line: $OA = 1$, $AB = 1$ perpendicular, $OB = \sqrt{2}$, arc from O ; square root spiral gives $\sqrt{(n + 1)}$
- Āryabhaṭa: $\pi \approx 3927/1250 = 3.1416$; Lambert (1761) proved π irrational; Mādhava: $\pi = 4(1 - 1/3 + 1/5 - \dots)$

- Real numbers R = rationals and irrationals together; they fill the number line
- Rational decimals terminate or repeat; terminate exactly when the lowest-form denominator is $2^m 5^n$
- Pure repeating: multiply by 10^n and subtract. General repeating: shift by 10^m , then by 10^n , subtract
- $0.999... = 1$; every terminating decimal has a second form ending in repeating 9s

Weightage and Exam Pattern

Class 9 has no board exam. The chapter is assessed in school exams: periodic tests, the half-yearly exam and the annual exam. In the CBSE Class IX Mathematics course structure for 2026-27, **Unit I, Number System, carries 7 marks** out of the 80-mark theory paper and is allotted 12 teaching periods. The remaining 20 marks are internal assessment (pen-paper tests and multiple assessment 10, portfolio 5, lab practical 5).

Unit	Marks (out of 80)
I. Number System	07
II. Algebra	20
III. Coordinate Geometry	04
IV. Geometry	25
V. Mensuration	14
VI. Statistics and Probability	10

The CBSE question paper design weights questions by typology: about 54% remembering and understanding (definitions, classifying numbers, the terminating test), 24% applying (rational arithmetic, rationals between two numbers, decimal conversion) and 22% analysing, evaluating and creating (irrationality proofs, reasoning about closure, density or decimal places).

The course structure lists these key concepts for the unit: introduction to rational numbers, their representation on the number line, density of rational numbers and its proof, finding rational numbers between two rational numbers, decimal representation of rational numbers, introduction to irrational numbers, the proof of irrationality of $\sqrt{2}$ and $\sqrt{3}$, and the square root spiral. This chapter is also the foundation for Class 10 Real Numbers, where the irrationality of $\sqrt{2}$, $\sqrt{3}$ and $\sqrt{5}$ is proved again using prime factorisation in the board year.

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