ChapterNotes

chapternotes.in · free NEET & JEE prep

CHAPTER NOTES

Vector Algebra Class 12 Notes

- ✓ Chapter-wise notes & important questions
- ✓ Free gamified practice app — streaks, levels, revision
- ✓ Free mock & prediction papers with answer keys

Source: https://chapternotes.in/cbse/class-12-maths/vector-algebra-class-12-notes/

Scan for free practice, mocks & more notes

Vector Algebra Class 12 Notes

Vector Algebra is a fundamental chapter in Class 12 Maths that introduces you to quantities having both magnitude and direction. From physics forces to computer graphics, vectors are everywhere. This chapter carries **4–6 marks** in CBSE board exams and builds the foundation for 3D Geometry.

Key Concepts

1. Scalars and Vectors

Scalar	Vector
Has only magnitude	Has both magnitude and direction
Examples: mass, temperature, speed	Examples: force, velocity, displacement
Denoted by ordinary letters	Denoted by letters with arrow (→) on top

2. Types of Vectors

Type	Definition	Example
Zero Vector ($\vec{0}$)	Magnitude = 0, arbitrary direction	Force on a stationary object at rest
Unit Vector ($\hat{a}$)	Magnitude = 1	$\hat{i}$, $\hat{j}$, $\hat{k}$ along x, y, z axes
Co-initial	Same starting point	Vectors from origin
Collinear	Parallel (same or opposite direction)	$\vec{a} = \lambda\vec{b}$
Equal	Same magnitude and direction	Free vectors that overlap
Negative	Same magnitude, opposite direction	$-\vec{a}$

3. Position Vector

If a point P has coordinates (x, y, z), then its position vector is:

$\vec{OP} = x\hat{i} + y\hat{j} + z\hat{k}$

Magnitude: $|\overrightarrow{OP}| = \sqrt{x^2 + y^2 + z^2}$

4. Addition of Vectors

Triangle Law: Place vectors head-to-tail; the resultant goes from the tail of the first to the head of the last.

Parallelogram Law: Place vectors at same point; diagonal of parallelogram = resultant.

If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, then:
$\vec{a} + \vec{b} = (a_1+b_1)\hat{i} + (a_2+b_2)\hat{j} + (a_3+b_3)\hat{k}$

5. Section Formula

The position vector of point dividing the line joining A($\vec{a}$) and B($\vec{b}$):

Internally in ratio m:n → $\vec{r} = \frac{m\vec{b} + n\vec{a}}{m+n}$
Externally in ratio m:n → $\vec{r} = \frac{m\vec{b} - n\vec{a}}{m-n}$
Midpoint: $\vec{r} = \frac{\vec{a} + \vec{b}}{2}$

6. Scalar (Dot) Product

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$

In component form: $\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$

$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$

💡 **Quick Check:** If $\vec{a} \cdot \vec{b} = 0$, then the vectors are **perpendicular** ($\theta = 90°$). If $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|$, they are **parallel** ($\theta = 0°$).

Properties of Dot Product

- Commutative: $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$
- Distributive: $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$
- $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$
- $\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$

7. Cross (Vector) Product

$\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin\theta \cdot \hat{n}$

where $\hat{n}$ is the unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ (right-hand rule).

In component form:

$\vec{a} \times \vec{b} = (a_2b_3 - a_3b_2)\hat{i} - (a_1b_3 - a_3b_1)\hat{j} + (a_1b_2 - a_2b_1)\hat{k}$

$|\vec{a} \times \vec{b}|$ = Area of parallelogram with adjacent sides $\vec{a}$ and $\vec{b}$

$\frac{1}{2}|\vec{a} \times \vec{b}|$ = Area of triangle

Properties of Cross Product

- NOT commutative: $\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$
- Distributive: $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$
- $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = \vec{0}$
- $\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{k} = \hat{i}$, $\hat{k} \times \hat{i} = \hat{j}$

💡 **Remember:** Dot product gives a **scalar** → useful for finding angles. Cross product gives a **vector** → useful for finding areas and perpendicular directions.

Important Definitions

Term	Definition
Vector	A quantity with both magnitude and direction
Unit Vector	Vector with magnitude 1; $\hat{a} = \vec{a}/\|\vec{a}\|$
Collinear Vectors	Vectors that lie along the same or parallel lines
Dot Product	Scalar product: $\vec{a} \cdot \vec{b} = \|\vec{a}\|\|\vec{b}\| \cos\theta$
Cross Product	Vector product: $\vec{a} \times \vec{b} = \|\vec{a}\|\|\vec{b}\| \sin\theta \cdot \hat{n}$
Projection	Projection of $\vec{a}$ on $\vec{b}$ = $(\vec{a} \cdot \vec{b})/\|\vec{b}\|$

Solved Examples — NCERT-Based

Example 1: Find the unit vector in the direction of $\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$

Solution:

$|\vec{a}| = \sqrt{(4 + 9 + 36)} = \sqrt{49} = 7$

$\hat{a} = \vec{a}/|\vec{a}| = (2\hat{i} - 3\hat{j} + 6\hat{k})/7 = \mathbf{(2/7)\hat{i} - (3/7)\hat{j} + (6/7)\hat{k}}$

Example 2: Find the angle between $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$

Solution:

$\vec{a} \cdot \vec{b} = (1)(1) + (1)(-1) + (1)(1) = 1$

$|\vec{a}| = \sqrt{3}$, $|\vec{b}| = \sqrt{3}$

$\cos\theta = 1/(\sqrt{3} \times \sqrt{3}) = 1/3$

$\boldsymbol{\theta = \cos^{-1}(1/3)}$

Example 3: Find $\vec{a} \times \vec{b}$ if $\vec{a} = 3\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$

Solution:

$\vec{a} \times \vec{b} = [(2)(2) - (-1)(-1)]\hat{i} - [(3)(2) - (-1)(1)]\hat{j} + [(3)(-1) - (2)(1)]\hat{k}$

$= (4 - 1)\hat{i} - (6 + 1)\hat{j} + (-3 - 2)\hat{k}$

$= \mathbf{3\hat{i} - 7\hat{j} - 5\hat{k}}$

Example 4: Find the area of the triangle with vertices A(1,1,2), B(2,3,5), C(1,5,5)

Solution:

$\vec{AB} = (1)\hat{i} + (2)\hat{j} + (3)\hat{k}$, $\vec{AC} = (0)\hat{i} + (4)\hat{j} + (3)\hat{k}$

$\vec{AB} \times \vec{AC} = (6-12)\hat{i} - (3-0)\hat{j} + (4-0)\hat{k} = -6\hat{i} - 3\hat{j} + 4\hat{k}$

$|\vec{AB} \times \vec{AC}| = \sqrt{(36 + 9 + 16)} = \sqrt{61}$

Area = $\frac{1}{2}\sqrt{61} \approx$ **3.9 sq units**

Important Questions for Board Exams

1 Mark Questions

1. Find the magnitude of vector $3\hat{i} - 2\hat{j} + 6\hat{k}$.
2. If $|\vec{a}| = 3$ and $|\vec{b}| = 4$, find $|\vec{a} \times \vec{b}|$ when $\theta = 90°$.
3. Write the unit vector along $\hat{i} + \hat{j}$.

2 Mark Questions

1. Find the projection of $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$.
2. Show that vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ are not parallel.
3. Find λ if vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 6\hat{j} + \lambda\hat{k}$ are collinear.

3 Mark Questions

1. If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, prove that $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$.
2. Find the area of a parallelogram whose adjacent sides are $\vec{a} = 3\hat{i} + \hat{j} + 4\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$.

5 Mark Questions

1. If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$, find $\vec{a} + \vec{b}$, and verify that $(\vec{a} + \vec{b}) \times \vec{c} = \vec{a} \times \vec{c} + \vec{b} \times \vec{c}$.
2. Prove using vectors that the diagonals of a rhombus bisect each other at right angles.

Quick Revision Points

- Vectors have magnitude AND direction; scalars have only magnitude
- Unit vector: $\hat{a} = \vec{a}/|\vec{a}|$ (divide by magnitude)
- Section formula: $(m\vec{b} + n\vec{a})/(m+n)$ for internal division
- Dot product → scalar, gives angle: $\cos\theta = (\vec{a} \cdot \vec{b})/(|\vec{a}||\vec{b}|)$

- Cross product → vector, gives area: |a→×b→| = area of parallelogram
- Perpendicular ⟹ dot product = 0
- Parallel ⟹ cross product = 0→
- Projection of a→ on b→ = (a→·b→)/|b→|
- Area of triangle = ½|a→ × b→|

← Ch 9: Differential Equations

Ch 11: Three Dimensional Geometry →