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CHAPTER NOTES

Trigonometric Functions Class 11 Notes | CBSE Maths Chapter 3

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Trigonometric Functions Class 11 Notes | CBSE Maths Chapter 3

Trigonometric Functions is Chapter 3 of CBSE Class 11 Maths — and it is the chapter that quietly powers half of your higher-maths syllabus. It takes the simple sin, cos, and tan you met in Class 10 and stretches them to *any* angle, links them through powerful identities, and teaches you to solve equations involving them. Get this right and Inverse Trigonometric Functions, Calculus, and most of the Physics you study become far easier.

By the end of these notes you will be able to convert between degrees and radians, find the sign of any trigonometric ratio in any quadrant, use the sum, difference, and multiple-angle formulae, and write the general solution of a trigonometric equation. This is a high-weightage chapter carrying roughly 8–10 marks in boards, a heavy hitter in JEE, and the foundation for almost all of Class 12 Maths.

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Key Concepts

1. Angles and Their Measurement

An **angle** is the amount of rotation of a ray about its starting point. If the rotation is anticlockwise the angle is **positive**; if clockwise it is **negative**.

Angles are measured in two systems: the **degree measure** (a right angle = 90°, and 1° = 60′, 1′ = 60″) and the **radian measure** used throughout higher maths.

Radian Measure

One **radian** is the angle subtended at the centre of a circle by an arc whose length equals the radius. For an arc of length *l* on a circle of radius *r*, the angle is:

$\theta = l / r$ (θ in radians)

- A full circle = 2π radians = 360°, so **π radians = 180°**.
- **1 radian = 180°/π ≈ 57°16′**, and **1° = π/180 radian ≈ 0.01746 rad**.

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2. Degree–Radian Conversion

The single relation **π radians = 180°** handles every conversion you will ever need.

- **Degrees to radians:** multiply by $\pi/180$. Example: $60° = 60 \times \pi/180 = \pi/3$.
- **Radians to degrees:** multiply by $180/\pi$. Example: $3\pi/4 = (3\pi/4) \times 180/\pi = 135°$.

Standard angles to memorise: $30° = \pi/6$, $45° = \pi/4$, $60° = \pi/3$, $90° = \pi/2$, $180° = \pi$, $270° = 3\pi/2$, $360° = 2\pi$.

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3. The Six Trigonometric Functions

Instead of restricting ourselves to a right triangle, we define the functions on a **unit circle** (radius 1). If P(x, y) is the point on the unit circle for angle θ, then $\cos\theta = x$ and $\sin\theta = y$.

[DIAGRAM: A unit circle centred at O. A point P(x, y) makes angle θ with the positive x-axis; the horizontal coordinate x = cos θ, the vertical coordinate y = sin θ.]

The other four follow from these two:

- **$\tan\theta = \sin\theta / \cos\theta$** ($\cos\theta \neq 0$)
- **$\cot\theta = \cos\theta / \sin\theta$** ($\sin\theta \neq 0$)
- **$\sec\theta = 1 / \cos\theta$** ($\cos\theta \neq 0$)
- **$\operatorname{cosec}\theta = 1 / \sin\theta$** ($\sin\theta \neq 0$)

Because P lies on the unit circle, **$-1 \le \sin\theta \le 1$** and **$-1 \le \cos\theta \le 1$** for every angle θ.

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4. Signs of Functions in the Four Quadrants

As P moves round the circle, x and y change sign, so the functions do too. The quick mnemonic is **"All Silver Tea Cups"** (A-S-T-C), reading anticlockwise from the first quadrant.

- **Quadrant I (0 to 90°): A**ll functions positive.
- **Quadrant II (90° to 180°):** only **S**in (and cosec) positive.
- **Quadrant III (180° to 270°):** only **T**an (and cot) positive.
- **Quadrant IV (270° to 360°):** only **C**os (and sec) positive.

Key idea: sin and cos repeat every 2π (period 2π); tan and cot repeat every π (period π).

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5. Domain and Range

Knowing the domain and range stops you writing impossible answers like "sin θ = 2".

Function	Domain	Range
$\sin\theta$	R (all reals)	$[-1, 1]$
$\cos\theta$	R	$[-1, 1]$
$\tan\theta$	$R - \{(2n+1)\pi/2\}$	R
$\cot\theta$	$R - \{n\pi\}$	R
$\sec\theta$	$R - \{(2n+1)\pi/2\}$	$R - (-1, 1)$
$\text{cosec}\,\theta$	$R - \{n\pi\}$	$R - (-1, 1)$

So sec θ and cosec θ are never strictly between −1 and 1 — a fact examiners love to test.

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6. Fundamental (Pythagorean) Identities

These three come straight from $x^2 + y^2 = 1$ on the unit circle and must be at your fingertips.

- $\sin^2\theta + \cos^2\theta = 1$
- $1 + \tan^2\theta = \sec^2\theta$
- $1 + \cot^2\theta = \text{cosec}^2\theta$

From these you can rewrite any one function in terms of another — the core skill behind simplification questions.

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7. Allied Angles and Periodicity

Angles like (−θ), (90° ± θ), (180° ± θ), (360° ± θ) are **allied angles**. Their ratios reduce to those of θ using two rules: fix the sign by the quadrant (ASTC), and switch the function (sin ↔ cos, tan ↔ cot, sec ↔ cosec) only when the reference angle is an odd multiple of 90° (i.e. 90° or 270°).

- $\sin(-\theta) = -\sin\theta$, $\cos(-\theta) = \cos\theta$, $\tan(-\theta) = -\tan\theta$ (cos is even; sin and tan are odd).
- $\sin(90° - \theta) = \cos\theta$, $\cos(90° - \theta) = \sin\theta$ (complementary angles).
- $\sin(180° - \theta) = \sin\theta$, $\cos(180° - \theta) = -\cos\theta$.

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8. Trigonometric Functions of Sum and Difference of Two Angles

These **compound-angle formulae** are the most-used results of the whole chapter.

- **$\sin(A + B) = \sin A \cos B + \cos A \sin B$**
- **$\sin(A - B) = \sin A \cos B - \cos A \sin B$**
- **$\cos(A + B) = \cos A \cos B - \sin A \sin B$**
- **$\cos(A - B) = \cos A \cos B + \sin A \sin B$**
- **$\tan(A + B) = (\tan A + \tan B) / (1 - \tan A \tan B)$**
- **$\tan(A - B) = (\tan A - \tan B) / (1 + \tan A \tan B)$**

Product-to-Sum and Sum-to-Product

- **$2 \sin A \cos B = \sin(A + B) + \sin(A - B)$**
- **$2 \cos A \sin B = \sin(A + B) - \sin(A - B)$**
- **$2 \cos A \cos B = \cos(A + B) + \cos(A - B)$**
- **$2 \sin A \sin B = \cos(A - B) - \cos(A + B)$**
- **$\sin C + \sin D = 2 \sin((C+D)/2) \cos((C-D)/2)$**
- **$\cos C + \cos D = 2 \cos((C+D)/2) \cos((C-D)/2)$**

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9. Multiple and Sub-Multiple Angle Formulae

Putting A = B in the compound formulae gives the **double-angle** results, and these in turn give the half-angle (sub-multiple) forms.

- **$\sin 2A = 2 \sin A \cos A = 2 \tan A / (1 + \tan^2 A)$**
- **$\cos 2A = \cos^2 A - \sin^2 A = 1 - 2 \sin^2 A = 2 \cos^2 A - 1 = (1 - \tan^2 A)/(1 + \tan^2 A)$**
- **$\tan 2A = 2 \tan A / (1 - \tan^2 A)$**

Triple-Angle Formulae

- **$\sin 3A = 3 \sin A - 4 \sin^3 A$**
- **$\cos 3A = 4 \cos^3 A - 3 \cos A$**
- **$\tan 3A = (3 \tan A - \tan^3 A) / (1 - 3 \tan^2 A)$**

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10. Trigonometric Equations and General Solutions

A **trigonometric equation** involves the trigonometric functions of an unknown angle. Because the functions are periodic, every such equation has **infinitely many** solutions — collected in a single **general solution** (n is any integer).

- **$\sin \theta = 0 \Rightarrow \theta = n\pi$**
- **$\cos \theta = 0 \Rightarrow \theta = (2n + 1)\pi/2$**
- **$\tan \theta = 0 \Rightarrow \theta = n\pi$**

- **$\sin\theta = \sin\alpha \Longrightarrow \theta = n\pi + (-1)^n \alpha$**
- **$\cos\theta = \cos\alpha \Longrightarrow \theta = 2n\pi \pm \alpha$**
- **$\tan\theta = \tan\alpha \Longrightarrow \theta = n\pi + \alpha$**

Tip: always reduce the equation to one of these standard forms before writing the general solution.

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11. Sine Rule and Cosine Rule

For any triangle ABC with sides a, b, c opposite to angles A, B, C, two results relate the sides and angles (useful in heights-and-distances and in Physics).

- **Sine rule:** $a/\sin A = b/\sin B = c/\sin C = 2R$, where R is the circumradius.
- **Cosine rule:** $\cos A = (b^2 + c^2 - a^2) / (2bc)$, and similarly for cos B and cos C.

The cosine rule is the triangle version of the Pythagoras theorem — it reduces to $a^2 = b^2 + c^2$ when $A = 90°$.

Weightage in Board & Entrance Exams

Exam	Typical Weightage	Most-Tested Areas
CBSE Board (Class 11)	8–10 marks	Identities, compound angles, general solutions, radian conversion
JEE Main / Advanced	2–4 questions	Trigonometric equations, multiple-angle identities, max/min of expressions
Other entrance (CUET etc.)	2–3 questions	Signs & quadrants, allied angles, simplification of identities

[TABLE: Question-type split — VSA (1–2 marks): conversions, signs, simple identities; SA (3 marks): proving identities, compound-angle evaluation; LA (5 marks): general solution of equations, sum-to-product proofs.]

Important Definitions & Formulae

Term / Formula	Statement
Radian	Angle subtended at the centre by an arc equal to the radius: $\theta = l/r$
Degree–radian link	π radians = 180°; 1° = $\pi/180$ rad
Pythagorean identity	$\sin^2\theta + \cos^2\theta = 1$
Secant identity	$1 + \tan^2\theta = \sec^2\theta$
Cosecant identity	$1 + \cot^2\theta = \text{cosec}^2\theta$
Sum formula (sine)	$\sin(A + B) = \sin A \cos B + \cos A \sin B$
Sum formula (cosine)	$\cos(A + B) = \cos A \cos B - \sin A \sin B$
Double angle (cosine)	$\cos 2A = 1 - 2\sin^2 A = 2\cos^2 A - 1$
General solution (sine)	$\sin\theta = \sin\alpha \Longrightarrow \theta = n\pi + (-1)^n \alpha$
General solution (cosine)	$\cos\theta = \cos\alpha \Longrightarrow \theta = 2n\pi \pm \alpha$

Solved Examples

Example 1

Convert 40°20′ into radian measure.

Answer: 40°20′ = $40\frac{1}{3}$° = 121/3 degrees. In radians = $(121/3) \times (\pi/180)$ = **$121\pi/540$ radian**.

Example 2

Find the value of sin 75°.

Answer: $\sin 75° = \sin(45° + 30°) = \sin 45° \cos 30° + \cos 45° \sin 30° = (1/\sqrt{2})(\sqrt{3}/2) + (1/\sqrt{2})(1/2) = (\sqrt{3} + 1)/(2\sqrt{2})$ = **$(\sqrt{6} + \sqrt{2})/4$**.

Example 3

If $\cos\theta = -3/5$ and θ lies in the third quadrant, find $\sin\theta$ and $\tan\theta$.

Answer: $\sin^2\theta = 1 - 9/25 = 16/25$, so $\sin\theta = \pm 4/5$. In Q-III sin is negative, so **$\sin\theta = -4/5$**. $\tan\theta = \sin\theta/\cos\theta = (-4/5)/(-3/5)$ = **$4/3$**.

Example 4

Prove that $(1 + \tan^2 A)/(1 + \cot^2 A) = \tan^2 A$.

Answer: LHS = $\sec^2 A / \operatorname{cosec}^2 A = (1/\cos^2 A)/(1/\sin^2 A) = \sin^2 A/\cos^2 A$ = **$\tan^2 A$** = RHS. Hence proved.

Example 5

Find the general solution of $\sin\theta = \sqrt{3}/2$.

Answer: $\sin\theta = \sin(\pi/3)$, so $\theta = n\pi + (-1)^n(\pi/3)$, where $n \in \mathbb{Z}$. **General solution: $\theta = n\pi + (-1)^n\,\pi/3$**.

Example 6

Solve $2\cos^2 x + 3\cos x - 1 = 0$ and write its general solution.

Answer: Treat it as a quadratic in cos x: $\cos x = (-3 \pm \sqrt{17})/4$. Only $(-3 + \sqrt{17})/4 \approx 0.28$ lies in $[-1, 1]$, so **$x = 2n\pi \pm \alpha$**, where $\alpha = \cos^{-1}[(-3 + \sqrt{17})/4]$ and $n \in \mathbb{Z}$.

Important Questions for Board Exams

1–2-Mark Questions (VSA)

1. Convert $5\pi/3$ radians into degree measure.
2. State the sign of $\cos\theta$ and $\tan\theta$ when θ lies in the second quadrant.
3. Find the value of $\tan(-1125°)$.
4. Write the general solution of $\cos\theta = 0$.
5. What is the maximum value of $3\sin\theta + 4\cos\theta$?

3-Mark Questions (SA)

1. Prove that $\sin^2(\pi/6) + \cos^2(\pi/3) - \tan^2(\pi/4) = -1/2$.
2. If $\sin A = 3/5$ and A is acute, find sin 2A, cos 2A and tan 2A.
3. Prove that $(\cos 7x + \cos 5x)/(\sin 7x - \sin 5x) = \cot x$.
4. Find the value of cos 15° using a suitable compound-angle formula.

5-Mark Questions (LA)

1. Prove that $\cos 2x = 1 - 2\sin^2 x$ and hence find cos 2x when $\sin x = 1/3$.
2. Find the general solution of the equation $2\cos^2\theta + 3\sin\theta = 0$.
3. Prove that $\sin 3A = 3\sin A - 4\sin^3 A$ and use it to evaluate sin 18°.

Quick Revision Points

- π radians = 180°; degrees → radians multiply by $\pi/180$, radians → degrees multiply by $180/\pi$
- On the unit circle $\cos\theta = x$, $\sin\theta = y$; hence $-1 \le \sin\theta, \cos\theta \le 1$
- Signs by quadrant: All–Silver–Tea–Cups (Q-I all +, Q-II sin +, Q-III tan +, Q-IV cos +)
- $\sin^2\theta + \cos^2\theta = 1$; $1 + \tan^2\theta = \sec^2\theta$; $1 + \cot^2\theta = \text{cosec}^2\theta$
- cos is even, sin and tan are odd: $\cos(-\theta) = \cos\theta$, $\sin(-\theta) = -\sin\theta$; period 2π (tan period π)
- $\sin(A \pm B)$, $\cos(A \pm B)$, $\tan(A \pm B)$ — the compound-angle backbone
- $\cos 2A = 1 - 2\sin^2 A = 2\cos^2 A - 1$; $\sin 2A = 2 \sin A \cos A$
- $\sin 3A = 3 \sin A - 4 \sin^3 A$; $\cos 3A = 4 \cos^3 A - 3 \cos A$
- General solutions: $\sin\theta = \sin\alpha \implies \theta = n\pi + (-1)^n\alpha$; $\cos\theta = \cos\alpha \implies \theta = 2n\pi \pm \alpha$; $\tan\theta = \tan\alpha \implies \theta = n\pi + \alpha$
- Sine rule $a/\sin A = 2R$; cosine rule $\cos A = (b^2 + c^2 - a^2)/2bc$

Next Chapter: Chapter 4 — Principle of Mathematical Induction