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ChapterNotes

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CHAPTER NOTES

Three Dimensional Geometry Class 12 Notes

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Three Dimensional Geometry Class 12 Notes

Three Dimensional Geometry extends your understanding of lines and planes from 2D to 3D space. This chapter is **very important for CBSE boards (6–8 marks)** and frequently tested in competitive exams. It uses concepts from Vector Algebra, so make sure you're comfortable with dot and cross products.

Key Concepts

1. Direction Cosines and Direction Ratios

If a line makes angles α, β, γ with the positive x, y, z axes respectively:

Direction Cosines: $l = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$

Relation: $l^2 + m^2 + n^2 = 1$

Direction Ratios: a, b, c (proportional to l, m, n)

$l = a/\sqrt{(a^2+b^2+c^2)}$, $m = b/\sqrt{(a^2+b^2+c^2)}$, $n = c/\sqrt{(a^2+b^2+c^2)}$

💡 **Remember:** Direction ratios are NOT unique (can multiply by any scalar), but direction cosines ARE unique (always satisfy $l^2 + m^2 + n^2 = 1$).

2. Equation of a Line in 3D

Vector Form

Line passing through point $A(\vec{a})$ and parallel to vector $\vec{b}$:

$\vec{r} = \vec{a} + \lambda\vec{b}$

Line through two points $A(\vec{a})$ and $B(\vec{b})$:

$\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$

Cartesian Form

Through point (x_1, y_1, z_1) with direction ratios a, b, c:

$(x - x_1)/a = (y - y_1)/b = (z - z_1)/c$

Through two points (x_1, y_1, z_1) and (x_2, y_2, z_2):

$$(x - x_1)/(x_2 - x_1) = (y - y_1)/(y_2 - y_1) = (z - z_1)/(z_2 - z_1)$$

3. Angle Between Two Lines

If lines have direction ratios (a_1, b_1, c_1) and (a_2, b_2, c_2):

$$\cos\theta = |a_1a_2 + b_1b_2 + c_1c_2| / (\sqrt{(a_1^2+b_1^2+c_1^2)} \times \sqrt{(a_2^2+b_2^2+c_2^2)})$$

Parallel: $a_1/a_2 = b_1/b_2 = c_1/c_2$

Perpendicular: $a_1a_2 + b_1b_2 + c_1c_2 = 0$

4. Shortest Distance Between Two Lines

Skew Lines (non-parallel, non-intersecting)

Lines: $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$

Shortest Distance $= |(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})| / |\vec{b_1} \times \vec{b_2}|$

If shortest distance = 0, the lines **intersect**.

Parallel Lines

Lines: $\vec{r} = \vec{a_1} + \lambda\vec{b}$ and $\vec{r} = \vec{a_2} + \mu\vec{b}$

Distance $= |\vec{b} \times (\vec{a_2} - \vec{a_1})| / |\vec{b}|$

5. Equation of a Plane

Different Forms

Form	Equation	When to Use
Normal form (vector)	$\vec{r} \cdot \hat{n} = d$	Given normal and distance from origin
General form	$ax + by + cz + d = 0$	Standard form; (a,b,c) is normal
Point-normal (vector)	$(\vec{r} - \vec{a}) \cdot \vec{n} = 0$	Given a point and normal
Point-normal (Cartesian)	$a(x-x_1) + b(y-y_1) + c(z-z_1) = 0$	Given point (x_1,y_1,z_1) and normal (a,b,c)
Intercept form	$x/a + y/b + z/c = 1$	Given x, y, z intercepts
Three-point form	Use determinant	Given 3 non-collinear points

6. Angle Between Two Planes

Planes: $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$

$$\cos\theta = |a_1a_2 + b_1b_2 + c_1c_2| / (\sqrt{(a_1^2+b_1^2+c_1^2)} \times \sqrt{(a_2^2+b_2^2+c_2^2)})$$

Parallel: $a_1/a_2 = b_1/b_2 = c_1/c_2$

Perpendicular: $a_1a_2 + b_1b_2 + c_1c_2 = 0$

7. Distance of a Point from a Plane

Distance of point (x_1, y_1, z_1) from plane $ax + by + cz + d = 0$:

$$D = |ax_1 + by_1 + cz_1 + d| / \sqrt{(a^2 + b^2 + c^2)}$$

8. Angle Between a Line and a Plane

Line: $\vec{r} = \vec{a} + \lambda\vec{b}$, Plane: $\vec{r} \cdot \vec{n} = d$

$$\sin\phi = |\vec{b} \cdot \vec{n}| / (|\vec{b}| \times |\vec{n}|)$$

💡 **Key Difference:** Angle between two lines/planes uses **cos θ**, but angle between a line and a plane uses **sin φ**.

Important Definitions

Term	Definition
Direction Cosines	Cosines of angles a line makes with coordinate axes (l, m, n)
Skew Lines	Lines that are neither parallel nor intersecting
Normal to a Plane	Vector perpendicular to every line lying in the plane
Coplanar Lines	Lines that lie in the same plane (shortest distance = 0)

Solved Examples — NCERT-Based

Example 1: Find the equation of the line through (1, 2, 3) with direction ratios 4, 5, 6

Solution:

Cartesian form: $(x-1)/4 = (y-2)/5 = (z-3)/6$

Vector form: $\mathbf{\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(4\hat{i} + 5\hat{j} + 6\hat{k})}$

Example 2: Find the angle between planes 2x + y – 2z = 5 and 3x – 6y – 2z = 7

Solution:

$\vec{n_1} = (2, 1, -2)$, $\vec{n_2} = (3, -6, -2)$

$\cos\theta = |6 - 6 + 4| / (\sqrt{9} \times \sqrt{49}) = 4/21$

$\boldsymbol{\theta = \cos^{-1}(4/21)}$

Example 3: Find the distance of point (2, 5, –3) from the plane 6x – 3y + 2z – 4 = 0

Solution:

$D = |6(2) - 3(5) + 2(-3) - 4| / \sqrt{36 + 9 + 4}$

$= |12 - 15 - 6 - 4| / \sqrt{49} = |-13|/7 =$ **13/7 units**

Example 4: Find the shortest distance between the lines: $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 3\hat{j} + 2\hat{k})$ and $\vec{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(2\hat{i} + 3\hat{j} + \hat{k})$

Solution:

$\vec{a_1} = (1,2,3)$, $\vec{b_1} = (1,-3,2)$, $\vec{a_2} = (4,5,6)$, $\vec{b_2} = (2,3,1)$

$\vec{a_2} - \vec{a_1} = (3, 3, 3)$

$\vec{b_1} \times \vec{b_2} = (-3-6)\hat{i} - (1-4)\hat{j} + (3+6)\hat{k} = -9\hat{i} + 3\hat{j} + 9\hat{k}$

$|\vec{b_1} \times \vec{b_2}| = \sqrt{(81+9+81)} = \sqrt{171} = 3\sqrt{19}$

$(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2}) = -27 + 9 + 27 = 9$

SD $= |9|/(3\sqrt{19}) =$ **$3/\sqrt{19}$ units**

Important Questions for Board Exams

1 Mark Questions

1. Find the direction cosines of a line equally inclined to all three axes.
2. Write the equation of the plane with intercepts 2, 3, 4 on the axes.

2 Mark Questions

1. Find the distance of the point (3, −2, 1) from the plane $2x - y + 2z + 3 = 0$.
2. Find the angle between the lines with direction ratios (1, 1, 2) and ($\sqrt{3}-1$, $-\sqrt{3}-1$, 4).

3 Mark Questions

1. Find the equation of the plane passing through (1, 1, −1), (6, 4, −5), (−4, −2, 3).
2. Find the shortest distance between the lines $(x-1)/2 = (y-2)/3 = (z-3)/4$ and $(x-2)/3 = (y-4)/4 = (z-5)/5$.

5 Mark Questions

1. Find the equation of the plane passing through the intersection of planes $x + 3y + 6 = 0$ and $3x - y - 4z = 0$ whose perpendicular distance from origin is unity.

2. Find the foot of perpendicular and the perpendicular distance of the point (1, 2, 3) from the line $(x-6)/3 = (y-7)/2 = (z-7)/(-2)$. Also find the image of the point.

Quick Revision Points

- Direction cosines: $l^2 + m^2 + n^2 = 1$ (always!)
- Line equation: $\vec{r} = \vec{a} + \lambda\vec{b}$ or $(x-x_1)/a = (y-y_1)/b = (z-z_1)/c$
- Plane equation: $\vec{r} \cdot \vec{n} = d$ or $ax + by + cz + d = 0$
- Parallel lines $\Longrightarrow$ proportional direction ratios
- Perpendicular lines $\Longrightarrow a_1a_2 + b_1b_2 + c_1c_2 = 0$
- Distance from plane: $|ax_1+by_1+cz_1+d| / \sqrt{(a^2+b^2+c^2)}$
- Skew lines: use cross product formula for shortest distance
- Line-plane angle uses sin, plane-plane angle uses cos

← Ch 10: Vector Algebra

Ch 12: Linear Programming →