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System of Particles and Rotational Motion Class 11 Notes | CBSE Physics Chapter 6

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System of Particles and Rotational Motion Class 11 Notes | CBSE Physics Chapter 6

System of Particles and Rotational Motion is Chapter 6 of CBSE Class 11 Physics — the chapter that takes you from a single moving point to real, spinning, rolling bodies. It explains how to find the balance point (centre of mass) of any object, why a spinning skater speeds up when she pulls her arms in, and how a ball rolls down a slope. Master it and a huge slice of JEE and NEET mechanics becomes routine.

By the end of these notes you will be able to locate the centre of mass, use the vector (cross) product, write torque and angular momentum, apply conservation of angular momentum, calculate moment of inertia using the parallel and perpendicular axes theorems, and solve rolling-motion problems. This is a high-weightage chapter carrying roughly 6–8 marks in boards and very heavy weight in JEE — the rotational analogue of everything you learned in Laws of Motion.

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Key Concepts

1. Centre of Mass

The **centre of mass (CM)** of a body or system is the single point at which the whole mass can be taken to be concentrated for describing its translational motion. When you flip a spanner in the air, the CM follows a smooth parabola even though the spanner tumbles wildly.

For a system of particles, the position of the centre of mass is:

$$\mathbf{R} = (\mathbf{m}_1\mathbf{r}_1 + \mathbf{m}_2\mathbf{r}_2 + \dots + \mathbf{m}_n\mathbf{r}_n) / (\mathbf{m}_1 + \mathbf{m}_2 + \dots + \mathbf{m}_n) = \Sigma \mathbf{m}_i\mathbf{r}_i / M$$

- For a two-particle system on a line: $x_{\text{cm}} = (m_1x_1 + m_2x_2)/(m_1 + m_2)$.
 - For symmetric uniform bodies (sphere, ring, rod, disc), the CM lies at the geometric centre.
 - The CM may lie outside the body — for example, at the centre of a ring or a horseshoe.
-

2. Motion of the Centre of Mass

The centre of mass moves as if the total external force acted on the total mass concentrated there. Internal forces between particles cancel in pairs (Newton's third law) and never shift the CM.

$M \cdot a_{cm} = F_{external}$, and total momentum $P = M \cdot v_{cm}$

- If the net external force is zero, v_{cm} is constant — the CM moves with uniform velocity (or stays at rest).
 - This is why, when a shell explodes in mid-air, the fragments' CM keeps following the original parabolic path.
-

3. Vector (Cross) Product of Two Vectors

The **vector product** (or cross product) of two vectors gives a third vector perpendicular to both. It is the maths behind torque and angular momentum.

$A \times B = AB \sin \theta \hat{n}$, where $\hat{n}$ is given by the right-hand rule.

- Magnitude: $|A \times B| = AB \sin \theta$ (equals the area of the parallelogram formed by A and B).
 - It is **anti-commutative**: $A \times B = -(B \times A)$.
 - $\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{k} = \hat{i}$, $\hat{k} \times \hat{i} = \hat{j}$; and $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$.
 - If A and B are parallel ($\theta = 0^\circ$), the cross product is zero.
-

4. Angular Velocity and Angular Acceleration

Angular velocity (ω) is the rate of change of angular displacement. It is a vector along the axis of rotation, given by the right-hand rule.

$\omega = d\theta/dt$, and linear velocity $v = \omega \times r$ (so $v = \omega r$ in magnitude).

Angular acceleration (α) is the rate of change of angular velocity:

$\alpha = d\omega/dt$

- SI units: ω in rad/s, α in rad/s².
 - All particles of a rigid body share the same ω and α , but different linear speeds (greater for points farther from the axis).
-

5. Torque (Moment of Force)

Torque is the turning effect of a force about an axis — the rotational analogue of force. A longer spanner turns a bolt more easily because torque depends on the distance from the axis.

$\tau = r \times F$, so $\tau = rF \sin \theta = F \times (\text{perpendicular distance})$

- SI unit: N·m. It is a vector quantity.

- Torque is maximum when the force is applied perpendicular to the position vector ($\theta = 90^\circ$).
 - The rotational form of Newton's second law is $\tau = I\alpha$.
-

6. Angular Momentum

Angular momentum (L) is the rotational analogue of linear momentum — the “quantity of rotation” a body has.

$L = r \times p = r \times mv$, and for a rigid body $L = I\omega$

- SI unit: $\text{kg}\cdot\text{m}^2/\text{s}$ (or $\text{J}\cdot\text{s}$). It is a vector quantity.
 - The relation between torque and angular momentum is $\tau = dL/dt$ — the rotational version of $F = dp/dt$.
-

7. Conservation of Angular Momentum

If the net external torque on a system is zero, its total angular momentum remains constant.

If $\tau_{\text{ext}} = 0$, then $L = I\omega = \text{constant}$, so $I_1\omega_1 = I_2\omega_2$

This is why a spinning ice-skater speeds up when she pulls her arms in: reducing I forces ω to increase so that $I\omega$ stays fixed. A diver curling into a ball to spin faster uses the same principle.

8. Equilibrium of a Rigid Body

A rigid body is in **mechanical equilibrium** when both its linear acceleration and angular acceleration are zero. This needs two conditions to be satisfied together.

- **Translational equilibrium:** the net external force is zero ($\Sigma F = 0$).
- **Rotational equilibrium:** the net external torque is zero ($\Sigma \tau = 0$).

A **couple** is a pair of equal, opposite, non-collinear forces; it produces pure rotation with zero net force. The **principle of moments** for a lever states: load $\times$ load arm = effort $\times$ effort arm.

9. Moment of Inertia

Moment of inertia (I) is the rotational analogue of mass — it measures a body's resistance to a change in its rotational motion. It depends not just on mass but on how that mass is distributed about the axis.

$$I = \Sigma m_i r_i^2 = \int r^2 dm$$

- SI unit: $\text{kg}\cdot\text{m}^2$. It is a scalar (for a fixed axis).
 - Rotational kinetic energy: $KE = \frac{1}{2}I\omega^2$.
 - The same body can have different I about different axes — mass farther from the axis means larger I .
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10. Radius of Gyration

The **radius of gyration (k)** is the distance from the axis at which the whole mass of the body can be assumed concentrated to give the same moment of inertia.

$$I = Mk^2, \text{ so } k = \sqrt{I/M}$$

- SI unit: metre. It depends on the axis of rotation and on mass distribution.
 - For a thin rod about its centre, $k = L/\sqrt{12}$; for a solid sphere about a diameter, $k = \sqrt{(2/5)} \cdot R$.
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11. Theorems of Perpendicular and Parallel Axes

These two theorems let you find the moment of inertia about a new axis from a known one — saving you long integrations in the exam.

Perpendicular Axes Theorem (for plane laminae only)

The moment of inertia of a planar body about an axis perpendicular to its plane equals the sum of its moments of inertia about two mutually perpendicular axes in the plane, meeting at that point.

$$I_z = I_x + I_y$$

Parallel Axes Theorem (for any body)

The moment of inertia about any axis equals the moment of inertia about a parallel axis through the centre of mass plus Md^2 , where d is the distance between the axes.

$$I = I_{cm} + Md^2$$

12. Moments of Inertia of Standard Bodies

You must memorise these standard results — they appear directly in numericals and rolling problems.

Body	Axis	Moment of Inertia
Thin rod (length L)	Through centre, $\perp$ to rod	$ML^2/12$
Thin rod (length L)	Through one end, $\perp$ to rod	$ML^2/3$
Ring (radius R)	Through centre, $\perp$ to plane	MR^2
Disc (radius R)	Through centre, $\perp$ to plane	$\frac{1}{2}MR^2$
Solid sphere (radius R)	About a diameter	$(2/5)MR^2$
Hollow sphere (radius R)	About a diameter	$(2/3)MR^2$
Solid cylinder (radius R)	About its own axis	$\frac{1}{2}MR^2$

13. Kinematics and Dynamics of Rotational Motion

Rotation about a fixed axis obeys equations identical in form to the linear equations of motion — just swap the linear quantities for their angular partners.

Linear	Rotational
$v = u + at$	$\omega = \omega_0 + \alpha t$
$s = ut + \frac{1}{2}at^2$	$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = u^2 + 2as$	$\omega^2 = \omega_0^2 + 2\alpha\theta$
$F = ma$	$\tau = I\alpha$
$P = Fv$	$P = \tau\omega$

Work done by a torque is $\mathbf{W} = \boldsymbol{\tau}\boldsymbol{\theta}$, and rotational power is $\mathbf{P} = \boldsymbol{\tau}\boldsymbol{\omega}$.

14. Rolling Motion

Rolling without slipping combines translation of the centre of mass with rotation about the CM. The contact point is momentarily at rest, which is the condition $\mathbf{v}_{\text{cm}} = \boldsymbol{\omega}\mathbf{R}$.

The total kinetic energy of a rolling body splits into two parts:

$$\mathbf{KE} = \frac{1}{2}M\mathbf{v}_{\text{cm}}^2 + \frac{1}{2}I\boldsymbol{\omega}^2 = \frac{1}{2}M\mathbf{v}^2(1 + k^2/R^2)$$

For a body rolling down an incline of angle θ without slipping, the acceleration is:

$$a = g \sin \theta / (1 + k^2/R^2)$$

- Smaller $k^2/R^2 \rightarrow$ larger acceleration. A solid sphere ($k^2/R^2 = 2/5$) beats a disc ($1/2$), which beats a ring (1).
- This is why a solid sphere reaches the bottom of a ramp first, regardless of mass or radius.

Weightage in Board & Entrance Exams

Exam	Typical Weightage	Most-Tested Areas
CBSE Board (Class 11)	6–8 marks	Centre of mass, torque, moment of inertia, conservation of angular momentum
JEE Main / Advanced	2–4 questions (heavy weight)	Rolling motion, moment of inertia (theorems), angular momentum, pulley with mass
NEET	2–3 questions	Moment of inertia of standard bodies, torque, rotational KE, conservation of L

[TABLE: Question-type split — VSA (1 mark): definitions, units, standard MI values; SA (2–3 marks): torque/angular momentum numericals, theorems of axes; LA (5 marks): conservation of angular momentum, rolling on an incline derivation.]

Important Definitions

Term	Definition
Centre of mass	Point where the total mass of a system is taken to be concentrated: $R = \Sigma m_i r_i / M$
Vector product	$A \times B = AB \sin \theta \hat{n}$ — a vector perpendicular to both, by the right-hand rule
Angular velocity	Rate of change of angular displacement: $\omega = d\theta/dt$ (a vector along the axis)
Torque	Turning effect of a force: $\tau = r \times F = rF \sin \theta$
Angular momentum	Rotational analogue of momentum: $L = r \times p = I\omega$
Conservation of angular momentum	$L = I\omega$ is constant when net external torque is zero
Moment of inertia	Resistance to change in rotation: $I = \Sigma m_i r_i^2$
Radius of gyration	Distance k where $I = Mk^2$, so $k = \sqrt{I/M}$
Parallel axes theorem	$I = I_{cm} + Md^2$
Rolling condition	Rolling without slipping: $v_{cm} = \omega R$

Solved Examples

Example 1

Two particles of masses 2 kg and 3 kg are placed at $x = 0$ m and $x = 5$ m. Find the position of their centre of mass.

Answer: $x_{cm} = (m_1 x_1 + m_2 x_2) / (m_1 + m_2) = (2 \times 0 + 3 \times 5) / (2 + 3) = 15/5 = \mathbf{3 \text{ m}}$ from the origin.

Example 2

A force $F = (2\hat{i} + 3\hat{j})$ N acts at a point $r = (4\hat{i} + \hat{j})$ m. Find the torque about the origin.

Answer: $\tau = r \times F = (4\hat{i} + \hat{j}) \times (2\hat{i} + 3\hat{j}) = (4 \times 3 - 1 \times 2)\hat{k} = (12 - 2)\hat{k} = \mathbf{10 \hat{k} \text{ N}\cdot\text{m}}$.

Example 3

A wheel of moment of inertia $0.5 \text{ kg}\cdot\text{m}^2$ rotates at 20 rad/s . Find its angular momentum and rotational kinetic energy.

Answer: $L = I\omega = 0.5 \times 20 = \mathbf{10 \text{ kg}\cdot\text{m}^2/\text{s}}$. $KE = \frac{1}{2}I\omega^2 = \frac{1}{2} \times 0.5 \times 20^2 = \mathbf{100 \text{ J}}$.

Example 4

A disc of mass 2 kg and radius 0.5 m rotates about its central axis. Find its moment of inertia and radius of gyration.

Answer: $I = \frac{1}{2}MR^2 = \frac{1}{2} \times 2 \times 0.5^2 = \mathbf{0.25 \text{ kg}\cdot\text{m}^2}$. $k = \sqrt{I/M} = \sqrt{(0.25/2)} = \sqrt{0.125} \approx \mathbf{0.354 \text{ m}}$.

Example 5

A skater spinning at 3 rad/s with $I = 6 \text{ kg}\cdot\text{m}^2$ pulls in her arms, reducing I to $2 \text{ kg}\cdot\text{m}^2$. Find her new angular speed.

Answer: By conservation of angular momentum, $I_1\omega_1 = I_2\omega_2$. $\omega_2 = (6 \times 3)/2 = 18/2 = \mathbf{9 \text{ rad/s}}$.

Example 6

A solid sphere rolls without slipping down an incline of angle 30° . Find its acceleration. ($g = 10 \text{ m/s}^2$, $k^2/R^2 = 2/5$)

Answer: $a = g \sin \theta / (1 + k^2/R^2) = (10 \times 0.5) / (1 + 2/5) = 5 / (7/5) = 25/7 \approx \mathbf{3.57 \text{ m/s}^2}$.

Important Questions for Board Exams

1-Mark Questions (VSA)

1. Define moment of inertia and give its SI unit.
2. Why does a spinning skater speed up when she folds her arms in?
3. Can the centre of mass of a body lie outside the body? Give one example.
4. What is the value of $\hat{i} \times \hat{j}$?
5. State the rolling-without-slipping condition.

2–3-Mark Questions (SA)

1. State and explain the perpendicular axes theorem. To which bodies does it apply?
2. Define torque and angular momentum, and write the relation between them.
3. Derive the relation $\tau = I\alpha$ starting from the definition of torque.
4. A ring, a disc and a solid sphere of equal mass and radius roll down the same incline. Which reaches the bottom first and why?

5-Mark Questions (LA)

1. State the parallel axes theorem and use it to find the moment of inertia of a thin rod about an axis through one end perpendicular to its length.
2. State and prove the law of conservation of angular momentum from $\tau = dL/dt$, and explain two real-life applications.
3. Derive the expression for the acceleration of a body rolling without slipping down an inclined plane of angle θ .

Quick Revision Points

- Centre of mass: $R = \Sigma m_i r_i / M$; moves under net external force only (internal forces cancel)
- Total momentum $P = Mv_{cm}$; if $F_{ext} = 0$, v_{cm} is constant
- Vector product: $A \times B = AB \sin \theta \hat{n}$; anti-commutative; $\hat{i} \times \hat{j} = \hat{k}$
- Angular velocity $\omega = d\theta/dt$; $v = \omega \times r$; angular acceleration $\alpha = d\omega/dt$
- Torque $\tau = r \times F = rF \sin \theta$; rotational Newton's law $\tau = I\alpha$
- Angular momentum $L = r \times p = I\omega$; $\tau = dL/dt$
- Conservation of L: if $\tau_{ext} = 0$, then $I_1\omega_1 = I_2\omega_2$
- Moment of inertia $I = \Sigma m_i r_i^2$; radius of gyration $k = \sqrt{I/M}$; $KE = \frac{1}{2}I\omega^2$
- Perpendicular axes: $I_z = I_x + I_y$ (laminae); Parallel axes: $I = I_{cm} + Md^2$
- Standard MI: rod $ML^2/12$, ring MR^2 , disc $\frac{1}{2}MR^2$, solid sphere $(2/5)MR^2$, hollow sphere $(2/3)MR^2$
- Rolling: $v_{cm} = \omega R$; $KE = \frac{1}{2}Mv^2(1 + k^2/R^2)$; $a = g \sin \theta / (1 + k^2/R^2)$

Next Chapter: Chapter 7 — Gravitation



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