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Sequences and Series Class 11 Notes | CBSE Maths Chapter 8

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Sequences and Series Class 11 Notes | CBSE Maths

Chapter 8

Sequences and Series is Chapter 8 of CBSE Class 11 Maths — and one of the most scoring chapters in the whole syllabus. It takes a simple idea, numbers following a pattern, and turns it into powerful tools: arithmetic progressions, geometric progressions, and neat formulas for adding up long lists of numbers in a single step.

By the end of these notes you will be able to find any term of an AP or GP, sum any number of terms instantly, insert arithmetic and geometric means, sum an infinite GP, use the $AM \geq GM$ inequality, and apply the special-series formulas for Σn , Σn^2 , and Σn^3 . This is a high-weightage chapter carrying roughly 6–8 marks in boards, and the foundation for Binomial Theorem, Limits, and almost all of higher-class calculus.

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Key Concepts

1. Sequence and Series

A **sequence** is an ordered list of numbers arranged according to a definite rule. Each number is a **term**, written $a_1, a_2, a_3, \dots, a_n$, where a_n is the n th term (also called the general term).

For example, 2, 4, 6, 8, ... is a sequence whose rule is "add 2 each time". A sequence can be **finite** (a fixed number of terms) or **infinite** (terms go on forever).

A **series** is what you get when you add the terms of a sequence: $a_1 + a_2 + a_3 + \dots + a_n$. The sum of the first n terms is written as S_n , often using sigma notation $S_n = \sum_{k=1}^n a_k$ from $k = 1$ to n .

2. Arithmetic Progression (AP)

An **arithmetic progression** is a sequence in which each term differs from the previous one by a fixed number called the **common difference (d)**. So you get the next term by adding d every time.

Example: 3, 7, 11, 15, ... is an AP with first term $a = 3$ and common difference $d = 4$. A general AP looks like $a, a + d, a + 2d, a + 3d, \dots$

- **First term:** a
- **Common difference:** $d = a_n - a_{n-1}$ (constant for an AP)

nth Term of an AP

The n th term (general term) of an AP is:

$$a_n = a + (n - 1)d$$

The **n th term from the end** of an AP with last term ℓ is: $\ell - (n - 1)d$.

Sum of n Terms of an AP

The sum of the first n terms is:

$$S_n = (n/2)[2a + (n - 1)d]$$

If the last term ℓ is known, this simplifies to **$S_n = (n/2)(a + \ell)$** — just n times the average of the first and last terms.

Useful link: $a_n = S_n - S_{n-1}$, so you can recover any term from the sums.

3. Arithmetic Mean (AM)

If three numbers are in AP, the middle one is the **arithmetic mean** of the other two. So the AM of a and b is:

$$A = (a + b)/2$$

To insert **n arithmetic means** $A_1, A_2, \dots, A_n$ between a and b so that $a, A_1, \dots, A_n, b$ form an AP, the common difference is $d = (b - a)/(n + 1)$, and $A_k = a + k \cdot d$.

Key property: The sum of n AMs inserted between a and b equals n times the single AM of a and b , i.e. $n(a + b)/2$.

4. Geometric Progression (GP)

A **geometric progression** is a sequence in which each term is obtained by multiplying the previous one by a fixed non-zero number called the **common ratio (r)**.

Example: 2, 6, 18, 54, ... is a GP with first term $a = 2$ and common ratio $r = 3$. A general GP looks like $a, ar, ar^2, ar^3, \dots$

- **First term:** a ($a \neq 0$)
- **Common ratio:** $r = a_n/a_{n-1}$ (constant for a GP)

nth Term of a GP

The n th term (general term) of a GP is:

$$a_n = a \cdot r^{n-1}$$

Sum of n Terms of a GP

The sum of the first n terms of a GP (for $r \neq 1$) is:

$$S_n = a(r^n - 1)/(r - 1), \text{ used when } r > 1, \text{ and } S_n = a(1 - r^n)/(1 - r), \text{ used when } r < 1.$$

When $r = 1$, all terms are equal, so $S_n = na$.

5. Geometric Mean (GM)

If three numbers are in GP, the middle one is the **geometric mean** of the other two. The GM of two positive numbers a and b is:

$$G = \sqrt{ab}$$

To insert n **geometric means** $G_1, G_2, \dots, G_n$ between a and b so that $a, G_1, \dots, G_n, b$ form a GP, the common ratio is $r = (b/a)^{1/(n+1)}$, and $G_k = a \cdot r^k$.

Key property: The product of n GMs inserted between a and b equals the n th power of the single GM of a and b , i.e. $(\sqrt{ab})^n$.

6. Sum of an Infinite GP

If the common ratio satisfies $|r| < 1$, then $r^n \rightarrow 0$ as n grows large, so an infinite GP adds up to a finite value:

$$S_\infty = a/(1 - r), \text{ valid only for } -1 < r < 1.$$

For example, $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$ has $a = 1$ and $r = \frac{1}{2}$, so $S_\infty = 1/(1 - \frac{1}{2}) = 2$. If $|r| \geq 1$, the infinite sum does not exist (it grows without bound).

7. Relationship Between AM and GM

For any two **positive** numbers a and b , the arithmetic mean is always greater than or equal to the geometric mean:

$$A \geq G \Rightarrow (a + b)/2 \geq \sqrt{ab}$$

Equality holds only when $a = b$. This famous inequality ($AM \geq GM$) is a favourite in exams for proving maximum/minimum results.

Bonus relation: If A and G are the AM and GM of two numbers, those numbers are the roots of the quadratic $x^2 - 2Ax + G^2 = 0$, and they equal $A \pm \sqrt{A^2 - G^2}$.

8. Special Series (Sum of Natural Numbers, Squares, Cubes)

Some sums of powers of the first n natural numbers appear so often they are worth memorising.

- **Sum of first n natural numbers:** $\Sigma n = 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$
- **Sum of squares:** $\Sigma n^2 = 1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$
- **Sum of cubes:** $\Sigma n^3 = 1^3 + 2^3 + \dots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$

Neat fact: $\Sigma n^3 = (\Sigma n)^2$ — the sum of the first n cubes is exactly the square of the sum of the first n natural numbers.

To sum a series whose n th term is a polynomial in n , write the general term T_n , then add term by term using these standard results.

Weightage in Board & Entrance Exams

Exam	Typical Weightage	Most-Tested Areas
CBSE Board (Class 11)	6–8 marks	n th term & sum of AP/GP, AM/GM insertion, special series
JEE Main / Advanced	2–3 questions	Infinite GP, $AM \geq GM$ inequality, sum of special series
NDA / CUET / other	2–4 questions	AP & GP n th term, means, word problems

[TABLE: Question-type split — VSA (1 mark): find a term, common difference/ratio, AM/GM; SA (2–3 marks): sum of n terms, insert means, infinite GP; LA (4–5 marks): word problems, $AM \geq GM$ proofs, summing special series.]

Important Definitions & Formulae

Term / Result	Formula
nth term of AP	$a_n = a + (n - 1)d$
Sum of n terms of AP	$S_n = (n/2)[2a + (n - 1)d] = (n/2)(a + \ell)$
Arithmetic mean of a, b	$A = (a + b)/2$
nth term of GP	$a_n = a \cdot r^{n-1}$
Sum of n terms of GP ($r \neq 1$)	$S_n = a(r^n - 1)/(r - 1)$
Geometric mean of a, b	$G = \sqrt{ab}$
Sum of infinite GP ($ r < 1$)	$S_\infty = a/(1 - r)$
AM–GM inequality	$(a + b)/2 \geq \sqrt{ab}, a, b > 0$
Sum of first n natural numbers	$\Sigma n = n(n + 1)/2$
Sum of squares	$\Sigma n^2 = n(n + 1)(2n + 1)/6$
Sum of cubes	$\Sigma n^3 = [n(n + 1)/2]^2$

Solved Examples

Example 1

Find the 20th term of the AP: 5, 8, 11, 14, ...

Answer: Here $a = 5$, $d = 3$. $a_{20} = a + (n - 1)d = 5 + (20 - 1)(3) = 5 + 57 = \mathbf{62}$.

Example 2

Find the sum of the first 25 terms of the AP: 7, 10, 13, ...

Answer: $a = 7$, $d = 3$, $n = 25$. $S_n = (n/2)[2a + (n - 1)d] = (25/2)[14 + 24(3)] = (25/2)(86) = \mathbf{1075}$.

Example 3

Insert three arithmetic means between 4 and 20.

Answer: Here $a = 4$, $b = 20$, $n = 3$, so $d = (b - a)/(n + 1) = 16/4 = 4$. The means are $4 + 4 = 8$, $8 + 4 = 12$, $12 + 4 = 16$. So the AMs are **8, 12, 16**.

Example 4

Find the 7th term and the sum of the first 7 terms of the GP: 3, 6, 12, ...

Answer: $a = 3$, $r = 2$. $a_7 = a \cdot r^{n-1} = 3 \cdot 2^6 = 3 \times 64 = \mathbf{192}$. $S_7 = a(r^n - 1)/(r - 1) = 3(2^7 - 1)/(2 - 1) = 3(127) = \mathbf{381}$.

Example 5

Find the sum of the infinite GP: $6 + 4 + 8/3 + \dots$

Answer: $a = 6$, $r = 4/6 = 2/3$ ($|r| < 1$). $S_\infty = a/(1 - r) = 6/(1 - 2/3) = 6/(1/3) = \mathbf{18}$.

Example 6

Find the sum: $1^2 + 2^2 + 3^2 + \dots + 12^2$.

Answer: $\Sigma n^2 = n(n+1)(2n+1)/6 = 12 \times 13 \times 25/6 = 3900/6 = \mathbf{650}$.

Example 7

The AM of two positive numbers is 10 and their GM is 8. Find the numbers.

Answer: The numbers are $A \pm \sqrt{A^2 - G^2} = 10 \pm \sqrt{100 - 64} = 10 \pm 6$, giving **16 and 4**.

Important Questions for Board Exams

1-Mark Questions (VSA)

1. Write the common difference of the AP: 2, 5, 8, 11, ...
2. Find the geometric mean of 4 and 9.
3. State the condition under which the sum of an infinite GP exists.
4. Write the formula for the sum of the first n natural numbers.
5. If the n th term of a sequence is $a_n = 3n - 1$, find a_5 .

2–3-Mark Questions (SA)

1. Find the sum of the first 20 terms of the AP whose first term is 2 and common difference is 5.
2. Insert two geometric means between 3 and 81.
3. Find the sum of the infinite GP: $8 + 4 + 2 + 1 + \dots$
4. If the AM and GM of two numbers are 13 and 12 respectively, find the numbers.
5. Find the sum: $1^3 + 2^3 + 3^3 + \dots + 10^3$.

4–5-Mark Questions (LA)

1. Derive the formula for the sum of the first n terms of an arithmetic progression.
 2. For any two positive numbers a and b , prove that $AM \geq GM$, i.e. $(a + b)/2 \geq \sqrt{ab}$.
 3. The sum of three numbers in GP is 35 and their product is 1000. Find the numbers.
 4. Find the sum to n terms of the series whose n th term is $n(n + 1)$, using the special-series formulas.
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Quick Revision Points

- Sequence = ordered list by a rule; series = sum of its terms ($S_n = \sum a_k$)
 - AP: common difference d ; n th term $a_n = a + (n - 1)d$
 - Sum of AP: $S_n = (n/2)[2a + (n - 1)d] = (n/2)(a + \ell)$
 - AM of $a, b = (a + b)/2$; insert n AMs with $d = (b - a)/(n + 1)$
 - GP: common ratio r ; n th term $a_n = a \cdot r^{n-1}$
 - Sum of GP ($r \neq 1$): $S_n = a(r^n - 1)/(r - 1)$
 - GM of $a, b = \sqrt{ab}$; product of n GMs $= (\sqrt{ab})^n$
 - Infinite GP ($|r| < 1$): $S_\infty = a/(1 - r)$
 - $AM \geq GM$ for positive numbers; equal only when $a = b$
 - $\sum n = n(n + 1)/2$; $\sum n^2 = n(n + 1)(2n + 1)/6$; $\sum n^3 = [n(n + 1)/2]^2$
 - $\sum n^3 = (\sum n)^2$ — sum of cubes equals square of sum of naturals
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Next Chapter: Chapter 9 — Straight Lines



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