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CHAPTER NOTES

Probability Class 12 Notes

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Probability Class 12 Notes

Probability is the final chapter in Class 12 Maths and one of the most important for CBSE boards (**6–8 marks**). Building on Class 11 basics, this chapter introduces conditional probability, Bayes' theorem, and probability distributions. These concepts are widely used in data science, machine learning, and everyday decision-making.

Key Concepts

1. Conditional Probability

The probability of event E given that event F has already occurred:

$P(E|F) = P(E \cap F) / P(F)$, provided $P(F) \neq 0$

Properties:

• $P(S|F) = 1$ (where S is sample space)

• $P(E'|F) = 1 - P(E|F)$

• $P((A \cup B)|F) = P(A|F) + P(B|F) - P(A \cap B|F)$

💡 **Think of it this way:** Conditional probability "shrinks" the sample space to F. We only look at outcomes within F and check how many of those also belong to E.

2. Multiplication Theorem

$P(A \cap B) = P(A) \times P(B|A) = P(B) \times P(A|B)$

For three events:

$P(A \cap B \cap C) = P(A) \times P(B|A) \times P(C|A \cap B)$

3. Independent Events

Events A and B are independent if:

$P(A \cap B) = P(A) \times P(B)$

Equivalently: $P(A|B) = P(A)$ and $P(B|A) = P(B)$

(Knowing one event doesn't change the probability of the other)

💡 **Independent ≠ Mutually Exclusive!**

• Mutually exclusive: $A \cap B = \varnothing$ (can't happen together)

• Independent: knowing one doesn't affect the other

• If A and B are mutually exclusive with $P(A) > 0$, $P(B) > 0$, they CANNOT be independent!

4. Total Probability Theorem

If $E_1, E_2, \ldots, E_n$ form a partition of sample space S (mutually exclusive and exhaustive), then for any event A:

$$P(A) = P(E_1)P(A|E_1) + P(E_2)P(A|E_2) + \ldots + P(E_n)P(A|E_n)$$

$$= \sum P(E_i) \times P(A|E_i)$$

5. Bayes' Theorem

$$P(E_i|A) = P(E_i) \times P(A|E_i) \,/\, \sum P(E_j) \times P(A|E_j)$$

In words: **Posterior = (Prior × Likelihood) / Evidence**

💡 **When to use Bayes' Theorem:** When you know the "forward" probabilities (cause → effect) and need to find the "reverse" probability (effect → cause). Example: Given that a product is defective, what's the probability it came from factory A?

6. Random Variable and Probability Distribution

A **random variable X** is a real-valued function on the sample space. It assigns a number to each outcome.

Probability Distribution: A table listing all possible values of X with their probabilities.

Requirements:

• $0 \le P(X = x_i) \le 1$ for each value

• $\boldsymbol{\sum P(X = x_i) = 1}$ (probabilities must sum to 1)

7. Mean and Variance of a Random Variable

Mean (Expected Value): $E(X) = \mu = \sum x_i \times P(x_i)$

Variance: $Var(X) = E(X^2) - [E(X)]^2 = \sum x_i^2 \times P(x_i) - \mu^2$

Standard Deviation: $\sigma = \sqrt{Var(X)}$

8. Bernoulli Trials and Binomial Distribution

Bernoulli Trial: An experiment with exactly two outcomes — Success (p) and Failure ($q = 1-p$)

Binomial Distribution: For n independent Bernoulli trials:
$\mathbf{P(X = r) = {}^nC_r \times p^r \times q^{(n-r)}}$

where $r = 0, 1, 2, \ldots, n$

Mean: $E(X) = np$
Variance: $Var(X) = npq$

💡 **Identifying Binomial Distribution:** Check these conditions:
1. Fixed number of trials (n)
2. Each trial has exactly 2 outcomes
3. Probability of success (p) is constant across trials
4. Trials are independent

Important Definitions

Term	Definition
Conditional Probability	P(A|B) — probability of A given B has occurred
Independent Events	Events where $P(A \cap B) = P(A) \times P(B)$
Partition of S	Events $E_1, E_2, \ldots, E_n$ that are mutually exclusive and exhaustive
Bayes' Theorem	Formula to find reverse conditional probability
Random Variable	Real-valued function defined on sample space
Binomial Distribution	Distribution for n independent Bernoulli trials

Solved Examples — NCERT-Based

Example 1: A fair die is rolled. If E = {1,3,5} and F = {2,3}, find P(E|F).

Solution:

$E \cap F = \{3\}$

$P(E \cap F) = 1/6$, $P(F) = 2/6 = 1/3$

$P(E|F) = P(E \cap F)/P(F) = (1/6)/(1/3) =$ **1/2**

Example 2 (Bayes' Theorem): Bag I has 3 red, 4 black balls. Bag II has 5 red, 6 black balls. One bag is chosen at random and a ball is drawn. If the ball is red, find the probability it came from Bag I.

Solution:

Let E_1 = Bag I chosen, E_2 = Bag II chosen, A = red ball drawn

$P(E_1) = 1/2$, $P(E_2) = 1/2$

$P(A|E_1) = 3/7$, $P(A|E_2) = 5/11$

By Bayes' theorem:

$P(E_1|A) = P(E_1)\times P(A|E_1) / [P(E_1)\times P(A|E_1) + P(E_2)\times P(A|E_2)]$

$= (1/2 \times 3/7) / (1/2 \times 3/7 + 1/2 \times 5/11)$

$= (3/14) / (3/14 + 5/22) = (3/14) / (33+35)/(154) = (3/14) / (68/154)$

$= (3/14) \times (154/68) = (3 \times 11)/68 =$ **33/68**

Example 3: Find the probability distribution of number of heads in 3 tosses of a fair coin.

Solution:

n = 3, p = 1/2, q = 1/2. X = number of heads (0, 1, 2, 3)

X	0	1	2	3
P(X)	${}^3C_0(1/2)^3 = 1/8$	${}^3C_1(1/2)^3 = 3/8$	${}^3C_2(1/2)^3 = 3/8$	${}^3C_3(1/2)^3 = 1/8$

Mean = np = 3 × 1/2 = **3/2**

Variance = npq = 3 × 1/2 × 1/2 = **3/4**

Example 4: The probability that a student passes maths is 2/3 and physics is 4/9. Assuming independence, find the probability of passing at least one subject.

Solution:

P(M) = 2/3, P(P) = 4/9

P(at least one) = 1 – P(none) = 1 – P(M') × P(P')

= 1 – (1/3)(5/9) = 1 – 5/27 = **22/27**

Important Questions for Board Exams

1 Mark Questions

1. If $P(A) = 0.6$, $P(B) = 0.3$ and $P(A \cap B) = 0.2$, find $P(A|B)$.
2. If A and B are independent events with $P(A) = 0.3$ and $P(B) = 0.4$, find $P(A \cap B)$.

2 Mark Questions

1. A couple has 2 children. Find the probability that both are boys given that at least one is a boy.
2. A die is thrown twice. Events A = "sum is 8" and B = "first throw is 4". Are A and B independent?

3 Mark Questions

1. Find the mean and variance of the number of heads in 4 tosses of a coin.
2. A and B independently try to solve a problem. P(A solves) = 1/3, P(B solves) = 1/4. Find the probability that the problem is solved.

5 Mark Questions

1. Three machines A, B, C produce 25%, 35%, 40% of a factory's output. Defect rates are 5%, 4%, 2% respectively. A randomly chosen item is defective. Find the probability it was produced by machine C. (Bayes' theorem)
2. A random variable X has the distribution: $P(X=0) = 3k^3$, $P(X=1) = 4k-10k^2$, $P(X=2) = 5k-1$. Find k, mean, and variance.

Quick Revision Points

- $P(A|B) = P(A\cap B)/P(B)$ — "reduce" the sample space to B
- Independent $\Rightarrow P(A\cap B) = P(A)\times P(B)$; do NOT confuse with mutually exclusive
- Bayes' theorem reverses conditional probability: effect → cause
- Total probability = weighted sum over partition
- Probability distribution: all P(X) must be between 0 and 1, and sum to 1
- Mean $E(X) = \sum x_i P(x_i)$, Variance $= E(X^2) - [E(X)]^2$
- Binomial: $P(X=r) = {}^nC_r\, p^r\, q^{n-r}$, Mean = np, Variance = npq
- Bayes' theorem problems are very common (5 marks) — practice the table method!