ChapterNotes

chapternotes.in · free NEET & JEE prep

CHAPTER NOTES

Linear Programming Class 12 Notes

✓ Chapter-wise notes & important questions

✓ Free gamified practice app — streaks, levels, revision

✓ Free mock & prediction papers with answer keys

Source: https://chapternotes.in/cbse/class-12-maths/linear-programming-class-12-notes/

Linear Programming Class 12 Notes

Linear Programming (LP) is an optimisation technique used to find the best outcome (maximum profit or minimum cost) given constraints. This chapter carries **5 marks** in CBSE boards and is one of the most scoring chapters — once you master the graphical method, you can solve any LP problem!

Key Concepts

1. What is Linear Programming?

It is a method to achieve the best outcome (maximum or minimum) in a mathematical model whose requirements are represented by linear relationships.

Term	Meaning	Example
Objective Function	The function to be maximised or minimised	$Z = 3x + 5y$ (profit function)
Decision Variables	Variables we control	x = units of A, y = units of B
Constraints	Limitations/restrictions (linear inequalities)	$2x + y \leq 100$ (machine hours)
Non-negativity	Variables can't be negative	$x \geq 0, y \geq 0$
Feasible Region	Set of all points satisfying all constraints	Shaded region on graph
Feasible Solution	Any point in the feasible region	(10, 20)
Optimal Solution	Feasible solution that optimises Z	The point giving max/min Z

2. Corner Point Method (Steps)

Step 1: Write the objective function and constraints from the problem
Step 2: Graph each constraint as an equation (line), shade the feasible region
Step 3: Find all corner points (vertices) of the feasible region
Step 4: Calculate Z at each corner point
Step 5: The corner point giving max/min Z is the optimal solution

💡 **Fundamental Theorem:** If the optimal solution exists, it occurs at a **corner point** (vertex) of the feasible region. You never need to check interior points!

3. Types of Feasible Regions

Type	Description	Optimal Solution
Bounded	Enclosed region (finite area)	Always exists (both max and min)
Unbounded	Region extends infinitely	May or may not exist — need verification

💡 **For unbounded regions:** If you get Z = M at a corner point for maximisation, check if the half-plane Z > M has any point in common with the feasible region. If NO common point exists, then M is the maximum. Otherwise, no maximum exists.

4. Types of LP Problems in CBSE

Type	Objective	Common Scenario
Manufacturing	Maximise profit	Units of products A and B
Diet Problem	Minimise cost	Amount of food items for nutrition
Transportation	Minimise cost	Shipping goods from factories to warehouses
Allocation	Maximise output	Distributing resources

5. Graphing Tips

- Convert inequality to equation to plot the line
- Use test point (0,0) to determine which side to shade
- Find intersection points by solving pairs of equations simultaneously
- Don't forget non-negativity constraints ($x \geq 0$, $y \geq 0$) — this restricts the region to the first quadrant

Important Definitions

Term	Definition
Linear Programming	Optimisation technique with linear objective function and linear constraints
Feasible Region	Set of all points satisfying all constraints simultaneously
Optimal Solution	A feasible solution at which the objective function has max or min value
Corner Point	Vertex of the feasible region where constraint boundaries intersect
Bounded Region	Feasible region that can be enclosed in a circle

Solved Examples — NCERT-Based

Example 1: Manufacturing Problem

A factory makes two products A and B. Each A requires 2 hrs on machine I and 1 hr on machine II. Each B requires 1 hr on machine I and 3 hrs on machine II. Machine I is available for 10 hrs, machine II for 12 hrs. Profit on A = ₹40, on B = ₹60. Find the maximum profit.

Solution:

Let x = units of A, y = units of B

Maximise $Z = 40x + 60y$

Subject to: $2x + y \leq 10$, $x + 3y \leq 12$, $x \geq 0$, $y \geq 0$

Corner points: (0,0), (5,0), (0,4), and intersection of $2x+y=10$ and $x+3y=12$

Solving: $2x+y=10 \rightarrow y=10-2x \rightarrow x+3(10-2x)=12 \rightarrow x+30-6x=12 \rightarrow -5x=-18 \rightarrow x=18/5=3.6$, $y=2.8$

Corner points and Z values:

Corner Point	$Z = 40x + 60y$
(0, 0)	0
(5, 0)	200
(3.6, 2.8)	144 + 168 = 312
(0, 4)	240

Maximum profit = ₹312 at (3.6, 2.8)

Example 2: Diet Problem

A diet requires at least 10 units of vitamin A and 12 units of vitamin B. Food I costs ₹6/unit and provides 2 units of A, 1 unit of B. Food II costs ₹10/unit and provides 1 unit of A, 4 units of B. Find minimum cost.

Solution:

Let x = units of Food I, y = units of Food II

Minimise $Z = 6x + 10y$

Subject to: $2x + y \geq 10$, $x + 4y \geq 12$, $x \geq 0$, $y \geq 0$

Corner points: (0, 10), (12, 0), intersection of $2x+y=10$ and $x+4y=12$

Solving: $y = 10-2x \rightarrow x + 4(10-2x) = 12 \rightarrow x + 40-8x = 12 \rightarrow -7x = -28 \rightarrow x=4, y=2$

Corner Point	$Z = 6x + 10y$
(0, 10)	100
(4, 2)	44
(12, 0)	72

Minimum cost = ₹44 at $x = 4$ units of Food I, $y = 2$ units of Food II

Example 3: Maximise $Z = 5x + 3y$ subject to $3x + 5y \leq 15$, $5x + 2y \leq 10$, $x \geq 0$, $y \geq 0$

Solution:

Corner points: (0,0), (2,0), (0,3)

Intersection: $3x+5y=15$ and $5x+2y=10$

From $5x+2y=10$: $y = (10-5x)/2$. Substituting: $3x + 5(10-5x)/2 = 15 \rightarrow 6x + 50-25x = 30 \rightarrow -19x = -20 \rightarrow x = 20/19$, $y = 45/19$

Corner Point	$Z = 5x + 3y$
(0, 0)	0
(2, 0)	10
(20/19, 45/19)	$100/19 + 135/19 = 235/19 \approx 12.37$
(0, 3)	9

Maximum $Z = 235/19 \approx 12.37$ at (20/19, 45/19)

Example 4: Minimise $Z = x + 2y$ subject to $2x + y \geq 3$, $x + 2y \geq 6$, $x \geq 0$, $y \geq 0$

Solution:

Corner points of unbounded feasible region: (6, 0), (0, 3)

Intersection: $2x+y=3$ and $x+2y=6 \rightarrow y=3-2x \rightarrow x+2(3-2x)=6 \rightarrow x+6-4x=6 \rightarrow x=0, y=3$

Also check: (6, 0)

Corner Point	$Z = x + 2y$
(6, 0)	6
(0, 3)	6

Minimum $Z = 6$ (achieved at both points and all points on the line segment joining them)

Important Questions for Board Exams

1 Mark Questions

1. Define a feasible region in an LP problem.
2. What is the corner point theorem?

2 Mark Questions

1. Draw the feasible region for: $x + y \leq 4$, $x \geq 0$, $y \geq 0$ and find its corner points.
2. If the feasible region is bounded with vertices (0,0), (4,0), (0,3), find max $Z = 7x + 4y$.

5 Mark Questions

1. Maximise $Z = 3x + 2y$ subject to $x + 2y \leq 10$, $3x + y \leq 15$, $x \geq 0$, $y \geq 0$. Graph the constraints and find optimal solution.
2. A dealer deals in two products A and B. He has ₹30,000 to invest and space for 20 items. Product A costs ₹2,500, B costs ₹500. He can sell A at ₹500 profit and B at ₹150 profit. Formulate the LP problem and solve graphically for maximum profit.

Quick Revision Points

- LP = optimise a linear function subject to linear constraints
- Always check corner points — optimal solution is always at a vertex
- Steps: Formulate → Graph → Find corners → Evaluate Z → Pick best
- Bounded region → max and min both exist
- Unbounded region → may need extra verification
- Non-negativity ($x \geq 0$, $y \geq 0$) restricts to first quadrant
- If Z has same value at two corners → all points on that edge are optimal
- CBSE mostly asks 5-mark graphical problems — practice drawing accurate graphs!

← Ch 11: Three Dimensional Geometry

Ch 13: Probability →