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Limits and Derivatives Class 11 Notes | CBSE Maths Chapter 12

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Limits and Derivatives Class 11 Notes | CBSE Maths Chapter 12

Limits and Derivatives is Chapter 12 of CBSE Class 11 Maths — and it is the single most important bridge in your whole syllabus. It quietly introduces *calculus*, the mathematics of change, by answering two questions: what value does a function approach near a point (a **limit**), and how fast is a function changing at a point (a **derivative**). Get comfortable here and all of Class 12 calculus — continuity, differentiation, integration, and applications — becomes far easier.

By the end of these notes you will be able to evaluate limits of polynomial, rational and trigonometric functions, use the standard limit results (including the famous $(\sin x)/x \rightarrow 1$), find a derivative from first principles, and differentiate polynomials and trig functions using the algebra of derivatives. This chapter carries roughly 6–8 marks in boards and is a guaranteed JEE topic, so it rewards every minute you put in.

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Key Concepts

1. Intuitive Idea of a Limit

A **limit** tells you the value a function $f(x)$ gets close to as x gets close to some number a — without caring what happens exactly at a . We write this as $\lim_{x \rightarrow a} f(x) = L$.

Think of walking towards a doorway: the limit is the doorway you are heading for, even if you never quite step through it. For example, as $x \rightarrow 2$, the function $f(x) = x + 3$ clearly approaches 5, so $\lim_{x \rightarrow 2} (x + 3) = 5$.

Key idea: A limit describes a *trend* near a point, not the value *at* the point. This is why limits can exist even where the function itself is undefined.

2. Left-Hand and Right-Hand Limits

You can approach a point a from two sides. The value approached from the left (x slightly less than a) is the **left-hand limit (LHL)**, written $\lim_{x \rightarrow a^-} f(x)$. From the right (x slightly more than a) it is the **right-hand limit (RHL)**, written $\lim_{x \rightarrow a^+} f(x)$.

The limit **exists only if LHL = RHL**. If the two sides disagree, the limit does not exist.

For example, for $f(x) = |x|/x$ at $x = 0$, the LHL is -1 and the RHL is $+1$, so $\lim_{x \rightarrow 0} f(x)$ does not exist.

3. Algebra of Limits

Limits combine in exactly the way you would hope. If $\lim_{x \rightarrow a} f(x) = L$ and $\lim_{x \rightarrow a} g(x) = M$, then:

- **Sum:** $\lim [f(x) + g(x)] = L + M$
- **Difference:** $\lim [f(x) - g(x)] = L - M$
- **Product:** $\lim [f(x) \cdot g(x)] = L \cdot M$
- **Quotient:** $\lim [f(x)/g(x)] = L/M$, provided $M \neq 0$
- **Scalar:** $\lim [k \cdot f(x)] = k \cdot L$, where k is a constant

These rules let you break a complicated limit into simple pieces and evaluate each separately.

4. Limits of Polynomial and Rational Functions

For a **polynomial** function, limits are wonderfully easy — just substitute. Since polynomials are continuous everywhere, $\lim_{x \rightarrow a} p(x) = p(a)$.

For a **rational** function $f(x) = p(x)/q(x)$, substitute first. If $q(a) \neq 0$, the limit is simply $p(a)/q(a)$.

If substitution gives the **indeterminate form 0/0**, do not panic — factorise and cancel the common factor (or rationalise), then substitute again.

Example: $\lim_{x \rightarrow 2} (x^2 - 4)/(x - 2) = \lim_{x \rightarrow 2} (x - 2)(x + 2)/(x - 2) = \lim_{x \rightarrow 2} (x + 2) = 4$.

5. Standard Limit Results (Memorise These)

A handful of standard results appear again and again. They are the workhorses of every limit question.

- $\lim_{x \rightarrow a} (x^n - a^n)/(x - a) = n \cdot a^{n-1}$ (the algebraic standard limit, valid for all rational n)
- $\lim_{x \rightarrow 0} (\sin x)/x = 1$ (x in radians)
- $\lim_{x \rightarrow 0} (\tan x)/x = 1$
- $\lim_{x \rightarrow 0} (1 - \cos x)/x = 0$

- $\lim_{x \rightarrow 0} (1 - \cos x)/x^2 = 1/2$

Caution: The trigonometric standard limits only hold when the angle is measured in *radians*. This is exactly why calculus uses radians everywhere.

6. Limits of Trigonometric Functions

For most trig limits, you reshape the expression until it matches a standard result. The trick is to multiply and divide so that every sin and tan sits over its own angle.

Example: $\lim_{x \rightarrow 0} (\sin 3x)/(\sin 5x) = \lim_{x \rightarrow 0} [(\sin 3x)/(3x) \cdot 3x] \div [(\sin 5x)/(5x) \cdot 5x] = (1 \cdot 3)/(1 \cdot 5) = 3/5$.

Key idea: Whenever you see $(\sin \theta)/\theta$ shape forming as $\theta \rightarrow 0$, replace it with 1.

7. Derivative — Rate of Change & Geometric Meaning

The **derivative** of a function measures its instantaneous rate of change. If $y = f(x)$, the derivative $f'(x)$ tells you how fast y changes for a tiny change in x at that exact point.

Geometric meaning: the derivative $f'(a)$ is the *slope of the tangent* to the curve $y = f(x)$ at the point $x = a$. A steep tangent means a large derivative; a flat tangent means the derivative is zero.

[DIAGRAM: A curve $y = f(x)$ with a secant line through $(a, f(a))$ and $(a+h, f(a+h))$; as $h \rightarrow 0$ the secant rotates into the tangent at $(a, f(a))$, whose slope is $f'(a)$.]

Physical meaning: if $s = f(t)$ is displacement, then ds/dt is velocity. The derivative is change made precise.

8. Derivative from First Principles (ab initio)

The formal definition of the derivative is built straight from the idea of a limit of the slope of a shrinking secant.

$$f'(x) = \lim_{h \rightarrow 0} [f(x+h) - f(x)] / h$$

This is called finding the derivative **from first principles** (or the ab-initio method). You compute $f(x+h)$, subtract $f(x)$, divide by h , and take the limit as $h \rightarrow 0$.

Example: For $f(x) = x^2$, $f'(x) = \lim_{h \rightarrow 0} [(x+h)^2 - x^2]/h = \lim_{h \rightarrow 0} (2xh + h^2)/h = \lim_{h \rightarrow 0} (2x + h) = 2x$.

9. Algebra of Derivatives

Once you know a few basic derivatives, these rules let you differentiate almost anything in this chapter. If $u = f(x)$ and $v = g(x)$:

- **Sum/Difference:** $(u \pm v)' = u' \pm v'$

- **Constant multiple:** $(k \cdot u)' = k \cdot u'$
- **Product Rule:** $(u \cdot v)' = u'v + uv'$
- **Quotient Rule:** $(u/v)' = (u'v - uv')/v^2$, where $v \neq 0$

Tip: A clean way to remember the product rule — "first times derivative of second, plus second times derivative of first."

10. Derivatives of Polynomials and Trigonometric Functions

The power rule and the trig derivatives are the standard results you will use in every problem.

Power Rule

$\frac{d}{dx} (x^n) = n \cdot x^{n-1}$ (valid for all rational n)

- $\frac{d}{dx} (\text{constant}) = 0$
- $\frac{d}{dx} (x) = 1$
- For a polynomial, differentiate term by term: $\frac{d}{dx} (a_n x^n + \dots + a_1 x + a_0) = n \cdot a_n x^{n-1} + \dots + a_1$

Trigonometric Derivatives

Function $f(x)$	Derivative $f'(x)$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x \cdot \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cdot \cot x$

Note: In the Class 11 syllabus, $\sin x$ and $\cos x$ derivatives are derived from first principles using the standard limits; the others follow from the quotient rule.

Weightage in Board & Entrance Exams

Exam	Typical Weightage	Most-Tested Areas
CBSE Board (Class 11)	6–8 marks	0/0 limits, trig standard limits, first principles, differentiation rules
JEE Main / Advanced	1–2 questions	Trig & algebraic limits, indeterminate forms, derivatives
Class 12 prerequisite	Foundational	Continuity, differentiability, all of differential calculus

[TABLE: Question-type split — VSA (1–2 marks): direct limits & basic derivatives; SA (3 marks): 0/0 limits by factorisation, trig limits, first principles of simple functions; LA (5 marks): product/quotient rule derivatives, first principles of $\sin x / \cos x$.]

Important Definitions & Formulae

Term / Result	Definition / Formula
Limit	Value $f(x)$ approaches as $x \rightarrow a$: $\lim_{x \rightarrow a} f(x) = L$
Existence of limit	Limit exists at a only if $LHL = RHL$
Algebraic standard limit	$\lim_{x \rightarrow a} (x^n - a^n)/(x - a) = n \cdot a^{n-1}$
Sine standard limit	$\lim_{x \rightarrow 0} (\sin x)/x = 1$ (x in radians)
Tangent standard limit	$\lim_{x \rightarrow 0} (\tan x)/x = 1$
Cosine standard limit	$\lim_{x \rightarrow 0} (1 - \cos x)/x^2 = 1/2$
Derivative (first principles)	$f'(x) = \lim_{h \rightarrow 0} [f(x+h) - f(x)]/h$
Power rule	$d/dx (x^n) = n \cdot x^{n-1}$
Product rule	$(uv)' = u'v + uv'$
Quotient rule	$(u/v)' = (u'v - uv')/v^2$
Key trig derivatives	$(\sin x)' = \cos x$; $(\cos x)' = -\sin x$; $(\tan x)' = \sec^2 x$

Solved Examples

Example 1

Evaluate $\lim_{x \rightarrow 3} (x^2 + 2x - 1)$.

Answer: Polynomial $\rightarrow$ substitute directly: $(3)^2 + 2(3) - 1 = 9 + 6 - 1 = \mathbf{14}$.

Example 2

Evaluate $\lim_{x \rightarrow 2} (x^2 - 4)/(x - 2)$.

Answer: Substitution gives $0/0$. Factorise: $(x - 2)(x + 2)/(x - 2) = x + 2$. So the limit $= 2 + 2 = \mathbf{4}$.

Example 3

Evaluate $\lim_{x \rightarrow 0} (\sin 4x)/x$.

Answer: Multiply and divide by 4: $\lim_{x \rightarrow 0} 4 \cdot (\sin 4x)/(4x) = 4 \times 1 = \mathbf{4}$.

Example 4

Use the standard limit to evaluate $\lim_{x \rightarrow 2} (x^5 - 32)/(x - 2)$.

Answer: This is the form $(x^n - a^n)/(x - a)$ with $n = 5$, $a = 2$: $= 5 \cdot 2^4 = 5 \times 16 = \mathbf{80}$.

Example 5

Find the derivative of $f(x) = x^2 + 3x$ from first principles.

Answer: $f'(x) = \lim_{h \rightarrow 0} [(x+h)^2 + 3(x+h) - (x^2 + 3x)]/h = \lim_{h \rightarrow 0} (2xh + h^2 + 3h)/h = \lim_{h \rightarrow 0} (2x + h + 3) = \mathbf{2x + 3}$.

Example 6

Differentiate $y = x^3 \cdot \sin x$.

Answer: Product rule with $u = x^3$, $v = \sin x$: $y' = u'v + uv' = 3x^2 \cdot \sin x + x^3 \cdot \cos x = \mathbf{x^2(3 \sin x + x \cos x)}$.

Example 7

Differentiate $y = (x + 1)/(x - 1)$.

Answer: Quotient rule, $u = x + 1$ ($u' = 1$), $v = x - 1$ ($v' = 1$): $y' = [1 \cdot (x-1) - (x+1) \cdot 1]/(x-1)^2 = (x-1-x-1)/(x-1)^2 = \mathbf{-2/(x-1)^2}$.

Important Questions for Board Exams

1–2-Mark Questions (VSA)

1. Evaluate $\lim_{x \rightarrow 1} (x^2 + 1)$.
2. State the value of $\lim_{x \rightarrow 0} (\sin x)/x$ and the condition on the angle.
3. Find $d/dx (x^7)$.
4. What is the geometric meaning of the derivative $f'(a)$?
5. When does the limit of a function at a point fail to exist?

3-Mark Questions (SA)

1. Evaluate $\lim_{x \rightarrow 3} (x^2 - 9)/(x - 3)$ by factorisation.
2. Evaluate $\lim_{x \rightarrow 0} (\sin 5x)/(\sin 2x)$.
3. Find the derivative of $f(x) = 1/x$ from first principles.
4. Differentiate $y = (2x + 3)/(3x - 2)$ using the quotient rule.

5-Mark Questions (LA)

1. Find the derivative of $\sin x$ from first principles, using the standard limit $\lim_{x \rightarrow 0} (\sin x)/x = 1$.
 2. Differentiate $y = x^4 \cdot \cos x$ by the product rule, and state $f'(0)$.
 3. Evaluate $\lim_{x \rightarrow 0} (1 - \cos x)/x^2$ and hence find $\lim_{x \rightarrow 0} (1 - \cos 2x)/x^2$.
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Quick Revision Points

- Limit = value $f(x)$ approaches near a ; need not equal $f(a)$
 - Limit exists only if $LHL = RHL$
 - Polynomial limit: just substitute, $\lim_{x \rightarrow a} p(x) = p(a)$
 - $0/0$ form $\rightarrow$ factorise/rationalise, cancel, then substitute
 - Standard: $\lim_{x \rightarrow a} (x^n - a^n)/(x - a) = n \cdot a^{n-1}$
 - Standard trig (radians): $(\sin x)/x \rightarrow 1$, $(\tan x)/x \rightarrow 1$, $(1 - \cos x)/x^2 \rightarrow 1/2$
 - Derivative = rate of change = slope of tangent
 - First principles: $f'(x) = \lim_{h \rightarrow 0} [f(x+h) - f(x)]/h$
 - Power rule: $d/dx (x^n) = n \cdot x^{n-1}$; constant $\rightarrow 0$
 - Product: $(uv)' = u'v + uv'$; Quotient: $(u/v)' = (u'v - uv')/v^2$
 - $(\sin x)' = \cos x$, $(\cos x)' = -\sin x$, $(\tan x)' = \sec^2 x$
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Next Chapter: Chapter 13 — Statistics



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