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CHAPTER NOTES

# Introduction to Three Dimensional Geometry Class 11 Notes | CBSE Maths Chapter 11

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# Introduction to Three Dimensional Geometry Class 11 Notes | CBSE Maths Chapter 11

Introduction to Three Dimensional Geometry is Chapter 11 of CBSE Class 11 Maths — the chapter where geometry leaves the flat page and enters real space. Until now every point you plotted lived on a 2D plane with just  $x$  and  $y$ . Here a third axis,  $z$ , joins in, and suddenly you can locate any point in a room, describe the corner of a box, or fix a star in the sky using just three numbers.

By the end of these notes you will be able to set up the coordinate axes and planes in 3D, write the coordinates of any point and name its octant, find the distance between two points, apply the section formula (internal and external), and locate the midpoint and centroid of a triangle. This is a scoring, formula-driven chapter carrying roughly 4–6 marks in boards and a sure question in JEE — almost pure substitution once you know the rules.

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## Key Concepts

### 1. The Three Coordinate Axes

To fix a point in space we need **three mutually perpendicular lines** meeting at a common point. These are the  $x$ -axis,  $y$ -axis and  $z$ -axis, and the common point is the **origin  $O$** .

Think of the corner of your room: two edges of the floor give the  $x$ -axis and  $y$ -axis, and the vertical edge going up the wall gives the  $z$ -axis. Each pair of axes is at right angles to the other two.

[DIAGRAM: Three mutually perpendicular axes —  $x$  and  $y$  in the horizontal plane,  $z$  pointing vertically upward, all meeting at origin  $O$ .]

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## 2. The Three Coordinate Planes

The three axes, taken two at a time, determine three planes called the **coordinate planes**. They divide all of space, just as the two axes divided the plane into quadrants.

- **XY-plane:** contains the x-axis and y-axis. Every point on it has  $z = 0$ .
- **YZ-plane:** contains the y-axis and z-axis. Every point on it has  $x = 0$ .
- **ZX-plane:** contains the z-axis and x-axis. Every point on it has  $y = 0$ .

**Key idea:** A point on an axis has its other two coordinates zero — e.g. a point on the x-axis is  $(x, 0, 0)$ .

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## 3. Coordinates of a Point in Space

The position of any point P in space is given by an **ordered triplet  $(x, y, z)$** , where x, y and z are its perpendicular distances from the YZ-, ZX- and XY-planes respectively.

- $x$  = distance of P from the YZ-plane (measured along the x-axis)
- $y$  = distance of P from the ZX-plane (measured along the y-axis)
- $z$  = distance of P from the XY-plane (measured along the z-axis)

The coordinates of the **origin** are  $(0, 0, 0)$ . The order matters:  $(1, 2, 3)$  and  $(3, 2, 1)$  are completely different points.

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## 4. Octants

In 2D the two axes split the plane into 4 quadrants. In 3D the **three coordinate planes split space into 8 parts called octants**, each carrying a fixed combination of signs for  $(x, y, z)$ .

[TABLE: Sign pattern of  $(x, y, z)$  in each of the eight octants.]

| Octant | x | y | z |
|--------|---|---|---|
| I      | + | + | + |
| II     | – | + | + |
| III    | – | – | + |
| IV     | + | – | + |
| V      | + | + | – |
| VI     | – | + | – |
| VII    | – | – | – |
| VIII   | + | – | – |

**Trick:** The first octant has all three coordinates positive. The sign of  $z$  tells you "above" (+) or "below" (–) the XY-plane.

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## 5. Distance Between Two Points

The straight-line distance between two points is found by extending the 2D distance formula to three coordinates. For  $P(x_1, y_1, z_1)$  and  $Q(x_2, y_2, z_2)$ :

$$PQ = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$$

- It is always positive (or zero if the points coincide).
- Distance of a point  $P(x, y, z)$  from the origin:  $OP = \sqrt{(x^2 + y^2 + z^2)}$ .

**Use it to prove shapes:** equal sides  $\rightarrow$  isosceles/equilateral;  $PQ^2 + QR^2 = PR^2 \rightarrow$  right angle; collinearity  $\rightarrow$  one distance equals the sum of the other two.

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## 6. Section Formula (Internal Division)

If a point  $R$  divides the join of  $P(x_1, y_1, z_1)$  and  $Q(x_2, y_2, z_2)$  **internally** in the ratio  $m : n$ , then  $R$  lies between  $P$  and  $Q$  and its coordinates are:

$$R = \left( \frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n} \right)$$

Remember the cross pattern:  $m$  goes with the *second* point and  $n$  goes with the *first*.

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## 7. Section Formula (External Division)

If  $R$  divides  $PQ$  **externally** in the ratio  $m : n$ , then  $R$  lies outside the segment (on the line, beyond  $P$  or  $Q$ ). Just replace  $n$  by  $-n$ :

$$R = \left( \frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n} \right)$$

**Note:** Internal and external division differ only by the sign of  $n$ . A negative ratio in the internal formula automatically gives external division.

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## 8. Midpoint Formula

The **midpoint** is the special case of internal division in the ratio  $1 : 1$ . For  $P(x_1, y_1, z_1)$  and  $Q(x_2, y_2, z_2)$ :

$$M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$$

Just average each coordinate. This is the fastest formula in the chapter and a common 1-mark question.

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## 9. Centroid of a Triangle

The **centroid** G is the point where the three medians of a triangle meet; it divides each median in the ratio 2 : 1. For vertices  $A(x_1, y_1, z_1)$ ,  $B(x_2, y_2, z_2)$  and  $C(x_3, y_3, z_3)$ :

$$G = ( (x_1 + x_2 + x_3)/3, (y_1 + y_2 + y_3)/3, (z_1 + z_2 + z_3)/3 )$$

Simply average the three vertices coordinate-by-coordinate. The centroid is always inside the triangle and is its centre of mass.

## Weightage in Board & Entrance Exams

| Exam                    | Typical Weightage | Most-Tested Areas                                     |
|-------------------------|-------------------|-------------------------------------------------------|
| CBSE Board (Class 11)   | 4–6 marks         | Distance formula, section formula, midpoint, centroid |
| JEE Main                | 1 question        | Distance, section formula, collinearity, octants      |
| Foundation for Class 12 | High              | Three Dimensional Geometry (lines & planes), Vectors  |

[TABLE: Question-type split — VSA (1 mark): octants, coordinates on axes/planes, midpoint; SA (2–3 marks): distance, section formula, collinearity; LA (4–5 marks): centroid, proving triangle type, ratio of division.]

## Important Definitions & Formulas

| Term                           | Formula / Definition                                                |
|--------------------------------|---------------------------------------------------------------------|
| Coordinates of a point         | Ordered triplet $(x, y, z)$ — distances from YZ-, ZX-, XY-planes    |
| XY-plane / YZ-plane / ZX-plane | $z = 0$ / $x = 0$ / $y = 0$ respectively                            |
| Octants                        | 8 regions of space made by the three coordinate planes              |
| Distance between two points    | $PQ = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$       |
| Distance from origin           | $OP = \sqrt{(x^2 + y^2 + z^2)}$                                     |
| Section formula (internal)     | $( (mx_2 + nx_1)/(m+n), (my_2 + ny_1)/(m+n), (mz_2 + nz_1)/(m+n) )$ |
| Section formula (external)     | $( (mx_2 - nx_1)/(m-n), (my_2 - ny_1)/(m-n), (mz_2 - nz_1)/(m-n) )$ |
| Midpoint                       | $( (x_1 + x_2)/2, (y_1 + y_2)/2, (z_1 + z_2)/2 )$                   |
| Centroid of a triangle         | $( (x_1 + x_2 + x_3)/3, (y_1 + y_2 + y_3)/3, (z_1 + z_2 + z_3)/3 )$ |

# Solved Examples

## Example 1

Name the octant in which the point  $(-2, 3, -5)$  lies.

**Answer:** Signs are  $(-, +, -)$ , which matches **Octant VI**.

## Example 2

Find the distance between the points  $P(1, -3, 4)$  and  $Q(4, 1, 8)$ .

**Answer:**  $PQ = \sqrt{[(4-1)^2 + (1-(-3))^2 + (8-4)^2]} = \sqrt{[3^2 + 4^2 + 4^2]} = \sqrt{[9 + 16 + 16]} = \sqrt{41} \approx \mathbf{6.40 \text{ units}}$ .

## Example 3

Find the coordinates of the point that divides the join of  $A(1, -2, 3)$  and  $B(3, 4, -5)$  internally in the ratio  $2 : 3$ .

**Answer:** Here  $m = 2$ ,  $n = 3$ .

$x = (2 \cdot 3 + 3 \cdot 1)/5 = 9/5$ ,  $y = (2 \cdot 4 + 3 \cdot (-2))/5 = 2/5$ ,  $z = (2 \cdot (-5) + 3 \cdot 3)/5 = -1/5$ .

The point is  **$(9/5, 2/5, -1/5)$** .

## Example 4

Find the midpoint of the segment joining  $(3, -1, 7)$  and  $(-5, 5, 1)$ .

**Answer:**  $M = ((3 + (-5))/2, (-1 + 5)/2, (7 + 1)/2) = \mathbf{(-1, 2, 4)}$ .

## Example 5

Find the centroid of the triangle with vertices  $A(0, 6, 2)$ ,  $B(4, -2, 3)$  and  $C(2, 5, 1)$ .

**Answer:**  $G = ((0+4+2)/3, (6+(-2)+5)/3, (2+3+1)/3) = (6/3, 9/3, 6/3) = \mathbf{(2, 3, 2)}$ .

## Example 6

Show that the points  $A(0, 7, 10)$ ,  $B(-1, 6, 6)$  and  $C(-4, 9, 6)$  form an isosceles right-angled triangle.

**Answer:**  $AB^2 = (-1)^2 + (-1)^2 + (-4)^2 = 18$ ;  $BC^2 = (-3)^2 + 3^2 + 0^2 = 18$ ;  $CA^2 = 4^2 + 2^2 + 4^2 = 36$ .

Since  $AB = BC$ , the triangle is **isosceles**; and  $AB^2 + BC^2 = 18 + 18 = 36 = CA^2$ , so by the converse of Pythagoras it is **right-angled** at B.

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# Important Questions for Board Exams

## 1-Mark Questions (VSA)

1. In which octant does the point  $(4, -2, -7)$  lie?
2. Write the coordinates of the foot of the perpendicular from  $P(3, 5, 8)$  to the  $XY$ -plane.
3. What is the  $y$ -coordinate of every point lying on the  $ZX$ -plane?
4. Find the midpoint of the segment joining  $(2, 3, 4)$  and  $(6, 7, 8)$ .
5. Write the distance of the point  $(3, 4, 12)$  from the origin.

## 2–3-Mark Questions (SA)

1. Find the distance between the points  $(-3, 7, 2)$  and  $(2, 4, -1)$ .
2. Find the ratio in which the  $YZ$ -plane divides the line segment joining  $(-2, 4, 7)$  and  $(3, -5, 8)$ .
3. Show that the points  $(-2, 3, 5)$ ,  $(1, 2, 3)$  and  $(7, 0, -1)$  are collinear.
4. Find the coordinates of the point dividing the join of  $(1, -2, 3)$  and  $(3, 4, -5)$  externally in the ratio  $1 : 2$ .

## 4–5-Mark Questions (LA)

1. Find the coordinates of the centroid of the triangle whose vertices are  $(3, -5, 7)$ ,  $(-1, 7, -6)$  and  $(1, 1, 2)$ .
  2. Prove that the points  $A(1, 2, 3)$ ,  $B(-1, -2, -1)$ ,  $C(2, 3, 2)$  and  $D(4, 7, 6)$  are the vertices of a parallelogram.
  3. Find the point on the  $y$ -axis which is equidistant from the points  $(3, 1, 2)$  and  $(5, 5, 2)$ .
- 

## Quick Revision Points

- A point in space needs three coordinates: an ordered triplet  $(x, y, z)$
  - Coordinate planes:  $XY$  ( $z = 0$ ),  $YZ$  ( $x = 0$ ),  $ZX$  ( $y = 0$ )
  - The three planes divide space into 8 octants; first octant is  $(+, +, +)$
  - Point on an axis has its other two coordinates zero
  - Distance:  $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
  - Distance from origin:  $OP = \sqrt{x^2 + y^2 + z^2}$
  - Internal section:  $((mx_2 + nx_1)/(m+n), \dots)$  —  $m$  with 2nd point,  $n$  with 1st
  - External section: replace  $n$  by  $-n$  in the internal formula
  - Midpoint: average of coordinates,  $((x_1+x_2)/2, \dots)$
  - Centroid:  $((x_1+x_2+x_3)/3, (y_1+y_2+y_3)/3, (z_1+z_2+z_3)/3)$
  - Use distances to test isosceles/equilateral, right-angle (Pythagoras), and collinearity
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*Next Chapter: Chapter 12 — Limits and Derivatives*



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