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Complex Numbers and Quadratic Equations Class 11 Notes | CBSE Maths Chapter 4

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Complex Numbers and Quadratic Equations Class 11 Notes | CBSE Maths Chapter 4

Complex Numbers and Quadratic Equations is Chapter 4 of CBSE Class 11 Maths — the chapter that finally answers a question that bugged you in earlier classes: what is the square root of a negative number? By introducing a single new symbol, *i* (where $i^2 = -1$), an entire new family of numbers opens up, and suddenly **every** quadratic equation has a solution.

By the end of these notes you will be able to simplify powers of i, perform algebra on complex numbers, find the modulus, conjugate and argument, plot numbers on the Argand plane, write a number in polar form, solve quadratics with complex roots, and even find the square root of a complex number. This is a scoring chapter carrying roughly 4–6 marks in boards and a near-guaranteed question in JEE — its concepts also power later topics in Maths and Physics.

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Key Concepts

1. The Need for Complex Numbers

In the set of real numbers, the equation $x^2 + 1 = 0$ has **no solution**, because no real number squared gives –1. Yet such equations appear naturally in algebra, physics, and engineering, so mathematicians extended the number system.

To handle the square root of negative numbers we define a new symbol **i**, called the **imaginary unit**, such that $\mathbf{i = \sqrt{(-1)}}$, i.e. $\mathbf{i^2 = -1}$. With i in hand, every quadratic equation becomes solvable — this is the whole point of the chapter.

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2. The Imaginary Unit i and Its Powers

The powers of i cycle through just four values, repeating every 4 steps. This pattern lets you simplify any power of i instantly.

- $\mathbf{i^1 = i}$

- $i^2 = -1$
- $i^3 = i^2 \cdot i = -i$
- $i^4 = (i^2)^2 = 1$

Trick: For any integer n, divide n by 4 and keep the remainder r; then $i^n = i^r$. For example $i^{101} = i^{(4\times25+1)} = i^1 = i$.

Caution: The rule $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ holds only when at least one of a, b is non-negative. So $\sqrt{-4} \times \sqrt{-9} \neq \sqrt{36}$. Correctly: $\sqrt{-4} \times \sqrt{-9} = (2i)(3i) = 6i^2 = -6$.

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3. Complex Numbers: Standard Form

A **complex number** is any number of the form **$z = a + ib$**, where a and b are real numbers and $i = \sqrt{-1}$.

- **$a = \mathrm{Re}(z)$** is the **real part**.
- **$b = \mathrm{Im}(z)$** is the **imaginary part**.
- If $b = 0$, z is purely real; if $a = 0$ (and $b \neq 0$), z is purely imaginary.

Equality: Two complex numbers $a + ib$ and $c + id$ are equal if and only if $a = c$ *and* $b = d$. So one complex equation gives you two real equations.

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4. Algebra of Complex Numbers

Complex numbers add, subtract, multiply and divide much like binomials in i, remembering always that $i^2 = -1$.

- **Addition:** $(a + ib) + (c + id) = (a + c) + i(b + d)$
- **Subtraction:** $(a + ib) - (c + id) = (a - c) + i(b - d)$
- **Multiplication:** $(a + ib)(c + id) = (ac - bd) + i(ad + bc)$

Division is done by multiplying numerator and denominator by the conjugate of the denominator:

$$(a + ib)/(c + id) = (a + ib)(c - id)/(c^2 + d^2)$$

The **multiplicative inverse** of $z = a + ib$ ($z \neq 0$) is **$z^{-1} = (a - ib)/(a^2 + b^2)$**.

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5. Conjugate of a Complex Number

The **conjugate** of $z = a + ib$ is **$\bar{z} = a - ib$** — just flip the sign of the imaginary part. Geometrically it is the mirror image of z across the real axis.

Key Properties

- $z + \bar{z} = 2a = 2\,\mathrm{Re}(z)$
- $z - \bar{z} = 2ib = 2i\,\mathrm{Im}(z)$

- $z \cdot \bar{z} = a^2 + b^2 = |z|^2$
- Conjugate of a sum/product = sum/product of conjugates.

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6. Modulus of a Complex Number

The **modulus** of $z = a + ib$ is its distance from the origin in the Argand plane.

$$|z| = \sqrt{(a^2 + b^2)}$$

- **SI-style note:** $|z| \geq 0$ and is a real number.
- $|z|^2 = z \cdot \bar{z}$
- $|z_1 z_2| = |z_1||z_2|$ and $|z_1/z_2| = |z_1|/|z_2|$ $(z_2 \neq 0)$

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7. Argand Plane and Geometrical Representation

Every complex number $z = a + ib$ can be plotted as the point (a, b) on a plane called the **Argand plane** (or complex plane), where the x-axis is the **real axis** and the y-axis is the **imaginary axis**.

[DIAGRAM: Argand plane — point P(a, b) marked; line OP of length $|z| = \sqrt{(a^2+b^2)}$ from origin O; angle θ between OP and the positive real axis is the argument.]

The distance OP from the origin to the point is the **modulus |z|**, and the angle the line OP makes with the positive real axis is the **argument θ**.

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8. Polar Representation and Argument

Using modulus $r = |z|$ and argument θ, any non-zero complex number can be written in **polar form**:

$\mathbf{z = r(\cos\theta + i\sin\theta)}$, where $r = \sqrt{(a^2 + b^2)}$, $a = r\cos\theta$, $b = r\sin\theta$.

The **argument** θ satisfies $\mathbf{\tan\theta = b/a}$. The value of θ lying in $(-\pi, \pi]$ is called the **principal argument**, arg(z).

Finding the Argument (Quadrant Matters)

- If z lies in the **first quadrant** $(a > 0, b > 0)$: $\theta = \tan^{-1}(b/a)$.
- If z lies in the **second quadrant** $(a < 0, b > 0)$: $\theta = \pi - \tan^{-1}(b/|a|)$.
- If z lies in the **third quadrant** $(a < 0, b < 0)$: $\theta = -\pi + \tan^{-1}(b/a)$.
- If z lies in the **fourth quadrant** $(a > 0, b < 0)$: $\theta = -\tan^{-1}(|b|/a)$.

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9. Quadratic Equations with Complex Roots

For the quadratic equation $\mathbf{ax^2 + bx + c = 0}$ $(a \neq 0)$ with real coefficients, the roots are given by the quadratic formula:

$$x = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$$

The quantity **$D = b^2 - 4ac$** is the **discriminant**. When $D < 0$, the square root involves $\sqrt{(-1)} = i$, so the roots are complex.

- If **$D < 0$**, the roots are a pair of **complex conjugates**: $x = [-b \pm i\sqrt{(4ac - b^2)}] / 2a$.
- Complex roots of a real-coefficient quadratic always occur in conjugate pairs.

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10. Square Root of a Complex Number

To find $\sqrt{(a + ib)}$, assume **$\sqrt{(a + ib)} = x + iy$**. Squaring gives $x^2 - y^2 = a$ and $2xy = b$. Solving these two equations gives x and y.

A quick standard result: for **$\sqrt{(a + ib)}$**,

$x = \pm\sqrt{[(|z| + a)/2]}$ and **$y = \pm\sqrt{[(|z| - a)/2]}$**, where $|z| = \sqrt{(a^2 + b^2)}$, choosing signs so that $2xy$ has the same sign as b.

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11. Fundamental Theorem of Algebra (Introduction)

The **Fundamental Theorem of Algebra** states that every polynomial equation of degree n ($n \geq 1$) with complex coefficients has **exactly n roots** in the set of complex numbers (counted with multiplicity).

This is the deep reason we needed complex numbers in the first place: once we allow them, no polynomial equation is ever “unsolvable.” A quadratic always has 2 roots, a cubic 3 roots, and so on.

Weightage in Board & Entrance Exams

Exam	Typical Weightage	Most-Tested Areas
CBSE Board (Class 11)	4–6 marks	Algebra of complex numbers, conjugate, modulus, quadratic with $D < 0$
JEE Main / Advanced	1–2 questions	Powers of i, polar form, argument, modulus properties
State CETs	1–2 questions	Conjugate, division, square root of a complex number

[TABLE: Question-type split — VSA (1 mark): powers of i, real/imaginary part, conjugate; SA (2–3 marks): division, modulus & argument, polar form; LA (4–5 marks): square root of a complex number, quadratics with complex roots.]

Important Definitions

Term	Definition
Imaginary unit (i)	The number with $i = \sqrt{(-1)}$, so $i^2 = -1$
Complex number	A number of the form $z = a + ib$, with a, b real
Real part / Imaginary part	$Re(z) = a$, $Im(z) = b$
Conjugate	$\bar{z} = a - ib$ (mirror of z across the real axis)
Modulus	$\lvert z\rvert = \sqrt{(a^2 + b^2)}$; distance of z from the origin
Argand plane	Plane representing $z = a + ib$ as the point (a, b)
Argument	Angle θ with the positive real axis: $\tan \theta = b/a$
Polar form	$z = r(\cos \theta + i \sin \theta)$, $r = \lvert z\rvert$
Discriminant	$D = b^2 - 4ac$; $D < 0$ gives complex roots
Fundamental Theorem of Algebra	A degree-n polynomial has exactly n complex roots

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Solved Examples

Example 1

Evaluate i^{107}.

Answer: $107 = 4 \times 26 + 3$, so $i^{107} = i^3 = \mathbf{-i}$.

Example 2

Express $(3 + 2i) + (-5 + 4i)$ in standard form.

Answer: $(3 - 5) + (2 + 4)i = \mathbf{-2 + 6i}$.

Example 3

Multiply $(2 + 3i)(1 - 4i)$.

Answer: $2 - 8i + 3i - 12i^2 = 2 - 5i + 12 = \mathbf{14 - 5i}$ (using $i^2 = -1$).

Example 4

Express $(1 + 2i)/(1 - i)$ in the form $a + ib$.

Answer: Multiply by the conjugate $(1 + i)$: $[(1 + 2i)(1 + i)] / [(1 - i)(1 + i)] = (1 + i + 2i + 2i^2)/(1 + 1) = (1 + 3i - 2)/2 = (-1 + 3i)/2 =$ **$-\frac{1}{2} + (3/2)i$**.

Example 5

Find the modulus and argument of $z = 1 + i\sqrt{3}$.

Answer: $|z| = \sqrt{(1^2 + (\sqrt{3})^2)} = \sqrt{(1 + 3)} =$ **2**. $\tan \theta = \sqrt{3}/1$, and z is in the first quadrant, so $\theta =$ **$\pi/3$**. Polar form: $z = 2(\cos \pi/3 + i \sin \pi/3)$.

Example 6

Solve the quadratic equation $x^2 + x + 1 = 0$.

Answer: Here $a = 1$, $b = 1$, $c = 1$, so $D = 1 - 4 = -3 < 0$. $x = [-1 \pm \sqrt{(-3)}]/2 = [-1 \pm i\sqrt{3}]/2$. Roots: **$x = -\frac{1}{2} + (\sqrt{3}/2)i$ and $x = -\frac{1}{2} - (\sqrt{3}/2)i$** (a conjugate pair).

Example 7

Find the square root of $3 + 4i$.

Answer: $|z| = \sqrt{(3^2 + 4^2)} = 5$. $x = \pm\sqrt{[(5 + 3)/2]} = \pm 2$, $y = \pm\sqrt{[(5 - 3)/2]} = \pm 1$. Since $b = 4 > 0$, x and y have the same sign, so $\sqrt{(3 + 4i)} =$ **$\pm(2 + i)$**.

Important Questions for Board Exams

1-Mark Questions (VSA)

1. Find the value of $i^9 + i^{19}$.
2. Write the conjugate of $3 - 4i$.
3. What is the real part of $(2 + i)^2$?
4. Find the modulus of $6 + 8i$.
5. For what value is a complex number purely imaginary?

2–3-Mark Questions (SA)

1. Express $(3 - 2i)/(2 + 3i)$ in the form $a + ib$.
2. Find the modulus and the principal argument of $z = -1 + i$.
3. If $z = 2 - 3i$, verify that $z \cdot \bar{z} = |z|^2$.

4. Convert $z = -\sqrt{3} + i$ into polar form.

4–5-Mark Questions (LA)

1. Find the square root of the complex number $-7 - 24i$.
2. Solve the quadratic equation $\sqrt{2}\, x^2 + x + \sqrt{2} = 0$ and express the roots in standard form.
3. Find the modulus and argument of $(1 + i)/(1 - i)$, then write it in polar form.

Quick Revision Points

- $i = \sqrt{(-1)}$, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$; powers of i repeat every 4 steps
- Complex number: $z = a + ib$, with $Re(z) = a$, $Im(z) = b$
- $\sqrt{a} \times \sqrt{b} = \sqrt{(ab)}$ fails when both a, b are negative
- Conjugate: $\bar{z} = a - ib$; $z \cdot \bar{z} = a^2 + b^2 = |z|^2$
- Modulus: $|z| = \sqrt{(a^2 + b^2)} \geq 0$
- Division: multiply numerator and denominator by the conjugate of the denominator
- Argand plane: $z = a + ib$ plotted as point (a, b)
- Polar form: $z = r(\cos \theta + i \sin \theta)$, $r = |z|$, $\tan \theta = b/a$
- Quadratic: $x = [-b \pm \sqrt{(b^2 - 4ac)}]/2a$; $D < 0 \Rightarrow$ complex conjugate roots
- Square root of $a + ib$: solve $x^2 - y^2 = a$ and $2xy = b$
- Fundamental Theorem of Algebra: a degree-n polynomial has exactly n complex roots

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Next Chapter: Chapter 5 — Linear Inequalities