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CHAPTER NOTES

Binomial Theorem Class 11 Notes | CBSE Maths Chapter 7

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Binomial Theorem Class 11 Notes | CBSE Maths Chapter 7

Binomial Theorem is Chapter 7 of CBSE Class 11 Maths — the elegant shortcut for expanding expressions like $(a + b)^5$ without multiplying the bracket out five times. Instead of grinding through tedious algebra, you use a single formula built on binomial coefficients (nCr) and Pascal's triangle. Master it and questions on the general term, middle term, and "term independent of x" become almost mechanical.

By the end of these notes you will be able to expand any $(a + b)^n$ for a positive integer n, write down the general term T_{r+1} instantly, locate the middle term, find the coefficient of a particular power, identify the term independent of x, and pin down the greatest binomial coefficient. This chapter carries roughly 5–6 marks in boards and is a guaranteed scorer in JEE — pure pattern, very little to memorise.

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Key Concepts

1. What is a Binomial?

A **binomial** is an algebraic expression with exactly two terms, such as (a + b), (x – 2), or (2x + 3y). The **Binomial Theorem** gives a quick rule to expand any positive integral power of a binomial, like $(a + b)^n$, without repeated multiplication.

For small powers you already know the pattern: $(a + b)^2 = a^2 + 2ab + b^2$ and $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$. The theorem generalises this to any n.

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2. Binomial Theorem for a Positive Integral Index

For any positive integer n, the expansion of $(a + b)^n$ is the sum of (n + 1) terms, each carrying a **binomial coefficient** nCr.

$(a + b)^n = nC_0\, a^n + nC_1\, a^{n-1}b + nC_2\, a^{n-2}b^2 + \ldots + nC_n\, b^n$

In compact summation form:

$(a + b)^n = \Sigma\, nCr \cdot a^{n-r} \cdot b^r \;\; (r = 0 \text{ to } n)$

where **$nCr = n! / [r!(n - r)!]$** is the binomial coefficient.

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3. Observations from the Expansion

- The total number of terms is **(n + 1)**.
- The power of **a decreases** from n to 0, while the power of **b increases** from 0 to n.
- In every term, the **sum of the exponents** of a and b is always n.
- The binomial coefficients nC_0, nC_1, ..., nC_n are **equidistant from the ends and equal** (since $nCr = nC_{n-r}$).

Useful Special Cases

- **$(1 + x)^n = nC_0 + nC_1x + nC_2x^2 + \ldots + nC_nx^n$**
- **$(a - b)^n = nC_0\, a^n - nC_1\, a^{n-1}b + nC_2\, a^{n-2}b^2 - \ldots + (-1)^n\, nC_n\, b^n$** (signs alternate).
- Putting a = b = 1 in $(a + b)^n$ gives **$nC_0 + nC_1 + \ldots + nC_n = 2^n$** (sum of all coefficients).

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4. Pascal's Triangle

Pascal's triangle is a triangular array in which the binomial coefficients of each expansion appear as a row. Each entry is the sum of the two entries directly above it.

[DIAGRAM: Pascal's triangle — row n = 0 is 1; n = 1 is 1 1; n = 2 is 1 2 1; n = 3 is 1 3 3 1; n = 4 is 1 4 6 4 1; n = 5 is 1 5 10 10 5 1. Each number is the sum of the two numbers above it.]

Row n directly gives the coefficients of $(a + b)^n$. For example, row 4 (1 4 6 4 1) gives $(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$. The construction relies on **Pascal's rule: $^nCr + {}^nCr_{-1} = {}^{n+1}Cr$**.

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5. General Term of the Expansion

The single most important tool in this chapter is the **general term**, written $T_{(r+1)}$ (the $(r + 1)^{th}$ term). Almost every exam question is solved by writing this term and choosing r.

$T_{r+1} = nCr \cdot a^{n-r} \cdot b^r$

- For the 1st term put r = 0, for the 2nd term r = 1, and so on.
- For $(1 + x)^n$ the general term simplifies to **$T_{r+1} = nCr \cdot x^r$**.

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6. Middle Term(s)

The position of the middle term depends on whether n is even or odd, because there are (n + 1) terms in total.

- **If n is even:** there is one middle term, the **$(n/2 + 1)^{th}$** term.
- **If n is odd:** there are two middle terms, the **$((n+1)/2)^{th}$** and **$((n+3)/2)^{th}$** terms.

To find a middle term, just substitute the correct value of r into the general term T_{r+1}.

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7. Coefficient of a Particular Term

To find the coefficient of a specific power, say x^m, write the general term, simplify the power of x, set its exponent equal to m, and solve for r.

Method: T_{r+1} = nCr · (power of x in terms of r) → equate exponent to m → solve for r → substitute back to read off the coefficient.

Note the difference between the **term** (which includes the variable) and its **coefficient** (the numerical part only).

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8. Term Independent of x

A **term independent of x** is the constant term — the one in which the net power of x is zero. It is found by the same method as above, but you set the exponent of x equal to **0**.

Method: Write T_{r+1}, collect the power of x as an expression in r, put that power = 0, solve for r, and substitute to get the constant term.

If the value of r comes out as a non-integer, then **no term independent of x exists** in that expansion.

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9. Greatest Binomial Coefficient

Among the coefficients nC_0, nC_1, ..., nC_n, the largest one is the **greatest binomial coefficient**, which always occurs at the middle term.

- **If n is even:** the greatest coefficient is **$nC_{(n/2)}$**.
- **If n is odd:** there are two equal greatest coefficients, **$nC_{((n-1)/2)} = nC_{((n+1)/2)}$**.

Caution: The greatest *binomial coefficient* (just nCr) is different from the *numerically greatest term* in an expansion, which also depends on the values of a and b.

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10. Simple Applications

The binomial theorem is handy for quick numerical estimates and divisibility proofs.

- **Approximation:** for small x, $(1 + x)^n \approx 1 + nx$, so a number like $(1.01)^5 \approx 1 + 5(0.01) = 1.05$.
- **Computing powers:** evaluate $(99)^4$ as $(100 - 1)^4$ using the expansion.
- **Divisibility / remainder:** write a number as $(\text{multiple} \pm 1)^n$ and expand to find the remainder, e.g. showing $6^n - 5n$ leaves remainder 1 on division by 25.

Weightage in Board & Entrance Exams

Exam	Typical Weightage	Most-Tested Areas
CBSE Board (Class 11)	5–6 marks	General term, middle term, coefficient of a term, independent term
JEE Main / Advanced	1–2 questions	Independent term, greatest coefficient, sum of coefficients, divisibility
Other entrances (CUET, etc.)	1–2 questions	Basic expansion, Pascal's triangle, nCr properties

[TABLE: Question-type split — VSA (1 mark): number of terms, value of nCr, sum of coefficients; SA (2–3 marks): general term, middle term, coefficient of a power; LA (4–5 marks): term independent of x, greatest coefficient, application/divisibility problems.]

Important Formulae & Definitions

Term	Formula / Definition
Binomial theorem	$(a + b)^n = \Sigma\ nCr\ a^{n-r}\ b^r$, r = 0 to n
Binomial coefficient	$nCr = n! / [r!(n - r)!]$
Number of terms	(n + 1) terms in the expansion of $(a + b)^n$
General term	$T_{r+1} = nCr\ a^{n-r}\ b^r$
Middle term (n even)	$(n/2 + 1)^{th}$ term
Middle terms (n odd)	$((n+1)/2)^{th}$ and $((n+3)/2)^{th}$ terms
Term independent of x	Term in which the net power of x equals 0
Pascal's rule	$nCr + nC_{r-1} = (n+1)Cr$
Sum of coefficients	$nC_0 + nC_1 + \ldots + nC_n = 2^n$
Symmetry property	$nCr = nC_{n-r}$

Solved Examples

Example 1

Expand $(x + 2)^4$ using the binomial theorem.

Answer: Coefficients from row 4 are 1, 4, 6, 4, 1. So $(x + 2)^4 = x^4 + 4x^3(2) + 6x^2(2^2) + 4x(2^3) + 2^4 =$ **$x^4 + 8x^3 + 24x^2 + 32x + 16$**.

Example 2

Find the general term in the expansion of $(2x - 3)^7$ and hence the 4th term.

Answer: $T_{r+1} = 7Cr\ (2x)^{7-r}\ (-3)^r$. For the 4th term, r = 3: $T_4 = 7C_3\ (2x)^4\ (-3)^3 = 35 \times 16x^4 \times (-27) =$ **$-15120\ x^4$**.

Example 3

Find the middle term in the expansion of $(x + y)^8$.

Answer: n = 8 is even, so there is one middle term, the (8/2 + 1) = 5th term. $T_5 = 8C_4\ x^4\ y^4 =$ **$70\ x^4y^4$**.

Example 4

Find the coefficient of x^5 in the expansion of $(x + 3)^8$.

Answer: $T_{r+1} = 8Cr\ x^{8-r}\ 3^r$. For x^5, $8 - r = 5 \Rightarrow r = 3$. Coefficient = $8C_3 \times 3^3 = 56 \times 27 =$ **1512**.

Example 5

Find the term independent of x in the expansion of $(x + 1/x)^6$.

Answer: $T_{r+1} = 6Cr\ x^{6-r}\ (1/x)^r = 6Cr\ x^{6-2r}$. For independence, $6 - 2r = 0 \Rightarrow r = 3$. Term = $6C_3 =$ **20**.

Example 6

Find the greatest binomial coefficient in the expansion of $(1 + x)^{10}$.

Answer: n = 10 is even, so the greatest coefficient is $10C_{(10/2)} = 10C_5 =$ **252**.

Important Questions for Board Exams

1-Mark Questions (VSA)

1. How many terms are there in the expansion of $(2a - 3b)^{12}$?
2. Write the value of the sum $nC_0 + nC_1 + nC_2 + \ldots + nC_n$.
3. What is the general term in the expansion of $(a + b)^n$?
4. State Pascal's rule connecting nCr, nCr_{-1} and $(n+1)Cr$.
5. Why are the coefficients nCr and nC_{n-r} always equal?

2–3-Mark Questions (SA)

1. Find the 5th term in the expansion of $(2x - y)^6$.
2. Find the middle term(s) in the expansion of $(x - 2y)^7$.
3. Find the coefficient of x^3 in the expansion of $(1 + 2x)^9$.
4. Find the term independent of x in the expansion of $(2x^2 - 1/x)^9$.

4–5-Mark Questions (LA)

1. Find the term independent of x in the expansion of $(3x^2 - 1/(2x))^9$ and state whether it exists.
2. Find the greatest binomial coefficient in $(1 + x)^{11}$ and explain why there are two equal greatest values.
3. Using the binomial theorem, show that $6^n - 5n$ always leaves remainder 1 when divided by 25.

Quick Revision Points

- $(a + b)^n = \Sigma\, nCr\, a^{n-r} b^r$ has $(n + 1)$ terms; exponents of a and b add up to n
- Binomial coefficient $nCr = n! / [r!(n - r)!]$; $nCr = nC_{n-r}$ (symmetry)
- Pascal's triangle: each row gives the coefficients; $nCr + nCr_{-1} = (n+1)Cr$
- General term: $T_{r+1} = nCr\, a^{n-r} b^r$ — the master tool for this chapter
- Middle term: n even → $(n/2 + 1)^{th}$ term; n odd → two middle terms
- Coefficient of x^m: equate the power of x in T_{r+1} to m, solve for r
- Term independent of x: set the power of x equal to 0; non-integer r ⇒ no such term
- Greatest binomial coefficient sits at the middle: $nC_{(n/2)}$ (n even)
- Sum of all coefficients = 2^n (put a = b = 1)
- Applications: approximations $(1 + x)^n \approx 1 + nx$, computing powers, divisibility proofs

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Next Chapter: Chapter 8 — Sequences and Series